Math 108, Section A01 Name Solutions
Summer, 2009
Exam 1

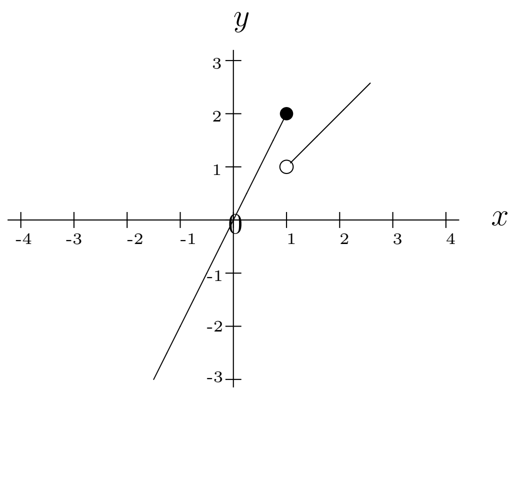
1. Let $y = f(x) = 6 - \frac{8}{x-2}$. Find $f(3)$, $f(1)$, and $f(2 + \frac{1}{x})$. If one of these expressions is not defined, write the words "Not defined" in the corresponding answer box.

$f(\ ) = 6 - \frac{8}{(\ )-2}$. Therefore, $f(3) = 6 - \frac{8}{3-2} = 6 - 8 = -2$, $f(1) = 6 - \frac{8}{1-2} = 6 + 8 = 14$, and $f(2 + \frac{1}{x}) = 6 - \frac{8}{(2+\frac{1}{x})-2} = 6 - 8x$.

$f(3) = \quad -2$
$f(1) = \quad 14$
$f(2 + \frac{1}{x}) = \quad 6 - 8x$

2. Let $y = f(x) = \begin{cases} 2x & \text{If } x \leq 1. \\ x & \text{If } x > 1. \end{cases}$ On the axes below, *carefully* draw the graph of f. Be clear about what the graph looks like at the point where $x = 1$.

For $x \leq 1$, the graph is the same as the graph of the line $y = 2x$. For $x > 1$, the graph is the same as the graph of the line $y = x$.



3. Let $y = f(x) = 3x^2 - 5x + 2$. Find the *exact* value of each x-intercept. If there are no x-intercepts, write "No x-intercepts" in the answer box.

The x-intercepts are the x-values where $3x^2-5x+2=0$. We can solve this equation by the quadratic formula, or we can notice that $3x^2-5x+2$ factors as $3x^2-5x+2=(3x-2)(x-1)$. Setting each factor equal to 0 gives that the x-intercepts are $x=\frac{2}{3}$ and $x=1$.

Answer:
$\frac{2}{3}$ and 1

4. Find the point(s) of intersection of the graphs of the functions $y = f(x) = 3x$ and $y = g(x) = \frac{24}{x+2}$.

We want to find where $f(x) = g(x)$, that is, where $3x = \frac{24}{x+2}$. If we multiply this equation by $x+2$, we get $3x(x+2) = 24$, or $3x^2+6x-24=0$. Dividing by 3 gives $x^2+2x-8=0$. The left side of this equation factors, so we can write the equation as $(x-2)(x+4)=0$. The solutions of this equation are $x = 2$ and $x = -4$. If $x = 2$, $y = f(2) = 6$. If $x = -4$, $y = f(-4) = -12$. Therefore, the points of intersection are $(2,6)$ and $(-4,-12)$.

Answer:
$(2,6)$ and $(-4,-12)$

5. A concert promoter finds that if tickets cost \$40 each, 3000 will be sold. For each \$2 increase in the ticket price, 50 fewer tickets will be sold. Express the revenue R from ticket sales as a function of the number x of \$2 increases in the ticket price.

The revenue R is the price per ticket times the number of tickets sold. The price per ticket (expressed in dollars) is \$40 plus \$2 for each of the x two-dollar increases, that is, $40+2x$. The number of tickets sold is 3000 minus 50 for each of the x two-dollar increases, that is, $3000-50x$. Therefore, $R(x) = (40+2x)(3000-50x) = 120000+4000x-100x^2$.

$R(x) = (40+2x)(3000-50x)$

6. Find an equation of the line through the points $(3, 8)$ and $(7, 6)$.

`The slope of the line is` $m = \frac{8-6}{3-7} = -\frac{1}{2}$. `Since the point` $(3, 8)$ `is on the line, an equation is given by` $y - 8 = (-\frac{1}{2})(x - 3)$. `This can be written as` $x+2y = 19$. `(There are other correct ways of writing the equation. For example, using the point` $(7, 6)$ `instead of` $(3, 8)$ `gives` $y-6 = (-\frac{1}{2})(x-7)$. `The slope-intercept form of the equation is` $y = (-\frac{1}{2})x + \frac{19}{2}$`.)`

Answer:
$x + 2y = 19$

7. Find the slope-intercept equation of the line with equation $6x - 2y = 5$.

`The equation can be written as` $2y = 6x - 5$. `Dividing by 2 gives the slope-intercept equation` $y = 3x - \frac{5}{2}$.

Answer:
$y = 3x - \frac{5}{2}$

In problems 8 through 10, find the indicated limit. If the limit does not exist, write "Does not exist" in the answer box.

8. $\lim_{x\to 3} \frac{x^2-4}{x^2+x-3}$

$\lim_{x\to 3} \frac{x^2-4}{x^2+x-3} = \frac{3^2-4}{3^2+3-3} = \frac{5}{9}$.

Answer:
$\frac{5}{9}$

9. $\lim_{x\to 3} \frac{x^2-9}{x^2-x-6}$

In this case, we cannot just replace the x by a 3 in the function because we would get 0 in the denominator. Since we also get 0 in the numerator, we can factor an $x-3$ from both the numerator and the denominator and cancel.

$\lim_{x\to 3} \frac{x^2-9}{x^2-x-6} = \lim_{x\to 3} \frac{(x-3)(x+3)}{(x-3)(x+2)} = \lim_{x\to 3} \frac{x+3}{x+2} = \frac{3+3}{3+2} = \frac{6}{5}$.

Answer:
$\frac{6}{5}$

10. $\lim_{x\to 3} f(x)$ where $f(x) = \begin{cases} 2x^2 - 4x & \text{If } x \neq 3. \\ 8 & \text{If } x = 3. \end{cases}$

In computing the limit of $f(x)$ as x approaches 3, the value of $f(3)$ is irrelevant. Therefore, $\lim_{x\to 3} f(x) = \lim_{x\to 3}(2x^2 - 4x) = 2(3^2) - 4(3) = 18 - 12 = 6$.

Answer:
6