ON THE SPECTRUM OF AN ISOMETRIC COMPOSITION OPERATOR ON THE BLOCH SPACE OF THE POLYDISK

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Abstract. Let f be a complex-valued holomorphic function defined on the unit polydisk $\mathbb{D}^n$. For $z \in \mathbb{D}^n$ and $u, v \in \mathbb{C}^n$, let

$$(\nabla f)(z)u = \langle (\nabla f)(z), \overline{u} \rangle,$$

where $(\nabla f)(z)$ is the gradient of f at z and

$$\langle u, v \rangle = \sum_{k=1}^{n} u_k \overline{v_k},$$

and let

$$H_z(u, \overline{v}) = \sum_{k=1}^{n} \frac{u_k \overline{v_k}}{(1 - |z_k|^2)^2}$$

be the Bergman metric on $\mathbb{D}^n$. The function f is said to be *Bloch* if $\beta_f = \sup_{z \in \mathbb{D}^n} Q_f(z) < \infty$ where

$$Q_f(z) = \sup_{u \in \mathbb{C}^n \setminus \{0\}} \frac{|(\nabla f)(z)u|}{H_z(u, \overline{u})^{1/2}}.$$

The set $\mathcal{B}$ of Bloch functions on $\mathbb{D}^n$ is a Banach space, called the *Bloch space*, under the norm $||f||_{\mathcal{B}} = |f(0)| + \beta_f$. Let φ be a holomorphic self-map on $\mathbb{D}^n$. In this talk, we study the spectrum of the bounded composition operator $C_\varphi : f \mapsto f \circ \varphi$ when this is an isometry on $\mathcal{B}$. We then give the complete description of the spectrum in the one-dimensional case. This is joint work with Flavia Colonna of George Mason University.