Math 722: Final Exam (Counts as two problem sets

Due on Friday, May 11, 11:45am

1. p. 79 #8.
2. p. 81 #21.
3. Consider the sequence of inclusions $S^{2k-1} \hookrightarrow S^{2k+1}$ induced by the inclusion of $\mathbb{C}^k \hookrightarrow \mathbb{C}^{k+1}$ into the first k coordinates of $\mathbb{C}^{k+1}$. You should think of theses spheres as the set of points distance one from the origin in the appropriate vector space.
   - Show that S^1 acts on S^{2k+1} by rotating each complex coordinate by $e^{2\pi i\theta}$. There are a few conditions that you need to check: that for each $\theta \in S^1$, this action produces a homeomorphism from S^{2k+1} to itself, that the identity acts by the identity homeomorphism, and that if $\theta, \rho \in S^1$, then the action by $\theta + \rho$ is the action of ρ followed by the action of θ.
   - Show that this inclusion is equivariant with respect to the action of S^1. In other words, show the inclusion commutes with the action.
   - Conclude that this induces and inclusion $\mathbb{C}P^{k-1} \to \mathbb{C}P^k$.
   - What is the induced map on homology?
4. Prove the *Generalized Jordan Curve Theorem*, which says If $A \subset S^n$ and $A \simeq S^k$, then

$$\tilde{H}_\ell(S^n \setminus A) \cong \begin{cases} \mathbb{Z} & \text{if } \ell \text{= n-k-1} \\ 0 & \text{otherwise} \end{cases}$$

5. Prove the *Lefschetz fixed point theorem for Δ complexes*. Let X be a finite Δ complex. Define the Lefschetz number of a map $f : X \to X$ by

$$L(f) = \sum_n (-1)^n Tr(f_* : H_n(X) \to H_n(X)),$$

   where the $Tr(f_* : H_n(X) \to H_n(X))$ is the trace of the TORSION FREE part (you can equivalently tensor $H_n(X)$ with $\mathbb{Q}$ and let f_* be the map on that tensor product induced from the map on $H_n(X)$, or you can just quotient out by any part of $H_n(X)$ that has torsion.) In other words, f_* is a map from a vector space of rank k to itself, so it has a trace when you write it in a basis.

   Theorem to prove: Suppose $f : X \to X$ has $L(f) \neq 0$. Then f has fixed points.

   Hint: Use barycentric subdivision! Also, use your text.

6. #2 p. 155
7. #9 p. 156