# MATH 351
## Problem Set 5
### Due October 25, 2012

1. Let $X$ be a Bernoulli random variable with parameter $p = \frac{5}{6}$. Find $E[cos(\pi X)]$, $E[3^X]$, and $E[\tan^{-1}(X)]$.

2. An urn contains 9 balls, 4 of which are red and 5 of which are blue. We draw a ball out of the urn 10 times, taking care to replace the ball and shake up the urn between draws. Let $X$ be the number of times that we draw a red ball.

   (a) What kind of random variable is $X$ and what are the parameters of the random variable?

   (b) What are $E[X]$ and $Var(X)$?

   (c) What is the probability that $X \leq 3$?

3. Suppose that $X$ is a binomial random variable with parameters $n$ and $p$. Find $E[X(X-1)(X-2)]$.

4. You have to pay \$100 to play the following game: A fair die is rolled until a 6 appears. If a 6 appears on the $n$th roll, you win $\left(\frac{6}{5}\right)^n$ dollars. The game finishes when a 6 appears. Let $X$ be your winnings from the game.

   (a) Prove that $E[X] = \infty$.

   (b) Would you pay a million dollars to play this game?

5. Let $X$ be a Poisson random variable with parameter $\lambda = 3$.

   (a) Find $P\{X > 1\}$

   (b) Find $E[X(X-1)(X-2)]$.

6. Compare the Poisson approximation with the correct binomial probability for the following cases:

   (a) $P\{X = 2\}$ when $n = 4, p = \frac{1}{2}$

   (b) $P\{X = 2\}$ when $n = 20, p = \frac{1}{10}$.

7. Suppose that a die is rolled until a 6 has appeared five times total (not necessarily in a row). Let $X$ be the number of the roll on which the fifth 6 appears.

(a) What kind of random variable is $X$? Make sure to specify any parameters.

(b) What is $E[X]$?

(c) What is $Var(X)$?