

**Math 351, Probability**  
**Problem Set 3**  
**Due September 20, 2012 in class**

1. If the letters of the word GOLDIN are arranged randomly in a row, what is the probability that none of the letters in the chosen word is in the same position in which it appears in GOLDIN?

*Hint.* Consider the event  $E_1$  that  $G$  is in the right place, the event  $E_2$  that  $O$  is in the right place, etc. How do you calculate the probability of each  $E_i$ ? Their union? What is the relationship between the probability you want to calculate and the probability that one of the  $E_i$  will happen?

2. Two cards are chosen at random from a deck of 52 cards. What is the probability that they

- (a) are from different suits?
- (b) are both 4s?

3. Find a formula for the number of functions from  $1, 2, \dots, k$  onto  $1, 2, \dots, n$ .

*Hint:* For  $i = 1, 2, \dots, n$ , let  $k_i$  be the number of functions that do not contain  $i$  in their ranges. Use inclusion-exclusion.

4. How many people need to be in a room in order that the probability that at least two of them have the same birthday is at least 0.8?
5. Compute the probability that a bridge hand (13 cards) has at least two cards from every suit.
6. A coin is flipped until heads has appeared four times. What is the probability that the fourth head appears on the tenth flip?
7. A blue die and a red die are rolled. What is the probability that
  - (a) their sum is at least 6, given that the red die is even?
  - (b) the red die is 3, given that their sum is at least 6?
8. Two cards are randomly chosen without replacement from an ordinary deck of 52 cards. Let  $B$  be the event that both cards are hearts. Let  $A$  be the event that the ace of hearts is chosen, and  $H$  the event that at least one heart is chosen.

- (a) Find  $P(B)$ .
- (b) Find  $P(B|A)$ .
- (c) Find  $P(B|H)$ .