MATH 351
Problem Set #2
due Tuesday, September 11 during class

1. Two dice are thrown. Let E be the event that the sum of the dice is even, let F be the event that at least one of the dice lands on 6 and let G be the event that the numbers on the two dice are equal. Find $P(E), P(F), P(G), P(E \cup F), P(EF), P(F \cup G), P(FG)$.

2. A card game requires each person to receive 4 cards from a standard deck of 52.

   (a) How many hands are possible?

   (b) What is the probability that a hand will contain 4 of a kind?

   (c) What is the probability that a hand will contain 3 of a kind?

3. Suppose we roll 4 6-sided dice simultaneously. Show that

   (a) $P(\text{no two alike}) = \frac{5}{18}$

   (b) $P(\text{two pair}) = \frac{5}{72}$

   (c) $P(3 \text{ alike}) = \frac{5}{54}$.

4. An urn contains 3 red, 5 blue and 7 green balls. A set of 3 of the balls is randomly selected.

   (a) What is the probability that all 3 of the selected balls are red?

   (b) What is the probability that all 3 have different colors?

5. A school offers Spanish, French and German language classes. 100 students take at least one of the three languages. 39 take Spanish, 38 take French, and 37 take German. 72 take Spanish or French, 2 take French and German, 4 take Spanish and French but not German.

   (a) How many students take all three languages?

   (b) How many take only Spanish?

6. A pair of dice is rolled until either **the two numbers on the dice agree** or **the difference of the two numbers on the dice is 1** (such as a 4 and a 5, or a 2 and a 1). Find the probability that you roll

two dice whose numbers agree before you roll two dice whose numbers differ by 1.

Hint: Let E_n denote the event that the numbers agree on the nth roll and that on the first $n-1$ rolls the dice neither agree nor differ by one. Compute $P(E_n)$ and argue that $\sum_{n=1}^{\infty} P(E_n)$ is the desired probability. Then calculate this sum.