MATH Practice 1 (for Exam 1)

Professor Goldin

1. Determine whether the following are statements. Write "STATEMENT" or "NOT STATEMENT" for each item.
   (a) Janis Joplin is cool and Justin Bieber is a dork. NOT STATEMENT
   (b) The final for Physics 104 at GMU is hard. NOT STATEMENT
   (c) Granite rock is harder than limestone. STATEMENT
   (d) This sentence is FALSE. NOT STATEMENT
   (e) If pigs fall from the sky, Dr. Goldin will give each student in Math 112 some money. STATEMENT.
2. Construct a truth table for the statement $(q \vee p) \rightarrow (p \wedge q) \vee \sim q$. ~~p∧q q∨p p∧q~~ (On next page)
3. Consider the following statements:

   $p =$ Jerry Jones has won the lottery.
   $q =$ Jerry Jones' wife filed for divorce.

   Write the English words for the symbolic statement.
   (a) $p \rightarrow \sim q$ If JJ has won the lottery, his wife has not filed for divorce.
   ✱ (b) $(p \wedge \sim q) \vee [(\sim p) \wedge q]$. (next page)
   (c) Write the symbolic statement for the English words, "If Jerry's wife did not file for divorce, then Jerry has won the lottery." $\sim q \longrightarrow p$
   (d) Write the symbolic statement for the English words, "Jerry has won the lottery only if his wife filed for divorce." $\sim q \rightarrow \sim p$
4. Prove the logical equivalence

$$p \rightarrow \sim q \Leftrightarrow \sim p \vee \sim q$$

5. Consider the statement, "You will get a million dollars in royalties if your book is a best-seller."
   (a) Write the contrapositive statement. If you didn't get a million dollars in royalty, then your book is not a best seller.

these are the same.

p	q	$\sim q$	$\boxed{p \rightarrow \sim q}$	$\sim p$	$\boxed{\sim p \vee \sim q}$
T	T	F	F	F	F
T	F	T	T	F	T
F	T	F	T	T	T
F	F	T	T	T	T

$\sim q$	q	p	$p \wedge q$	$(p \wedge q) \vee (\sim q)$
F	T	T	T	T
T	F	T	F	T
F	T	F	F	F
T	F	F	F	T

* ~~[illegible]~~

JJ won the lottery and his wife didn't file for divorce, or he lost the lottery and she did file for divorce.

(b) Write the converse statement. If you get a million in royalties, then your book is a best-seller.

6. Given that p is true and q, r are false, determine whether the following is TRUE, FALSE, or UNDETERMINED.

$$(p \wedge \sim q) \vee r.$$

T

7. Consider the universal set $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ and the open sentence

$$p(x) = \text{“}5x + 2 < 25\text{"}$$

Answer true or false.

T (a) $p(3)$. $5 \cdot 3 + 2 < 25$ ✓ T

F (b) $p(2) \wedge p(5)$. $p(5)$ F

F (c) $\forall x\, p(x)$.

T (d) $\exists x\, p(x)$.

T (e) $p(6) \rightarrow p(1)$. $p(6)$ F, so statement is true.

8. Use the laws of logical equivalences to negate the following statement: $(\sim p \vee q) \wedge q$

$\sim [(\sim p \vee q) \wedge q] \Leftrightarrow$

$\sim(\sim p \vee q) \vee \sim q \Leftrightarrow$

$(p \wedge \sim q) \vee \sim q$.

Solution not provided

9. Show the law of modus ponens holds:

$$p \wedge (p \rightarrow q) \Rightarrow q$$

10. Write a convincing argument that

$$(\sim [\forall x\, p(x)]) \Leftrightarrow (\exists x \sim p(x)).$$

11. Let $U = \{$all people in the United States.$\}$ Consider the statement

$$p(x, y) = \text{“}x \text{ is the mother of } y.\text{"}$$

Evaluate the truth value.

(a) $\forall y\, \exists x p(x, y)$ T (for each person, there is a mother)

(b) $\exists x\, \forall y p(x, y)$. F (There's a mother to all people)

12. Write an equivalent statement below without using the symbol $\rightarrow$. Simplify your answer at the end so it has no parentheses.

$$q \rightarrow p \wedge \sim r.$$

$q \vee (\sim [p \wedge \sim r])$

$q \vee (\sim p \vee r)$

$\boxed{q \vee \sim p \vee r.}$

13. True or False.

(a) A predicate is an example of a statement. F

(b) A syllogism is a logical conclusion following from two or more other statements. T

(c) The following is an example of a syllogism: T

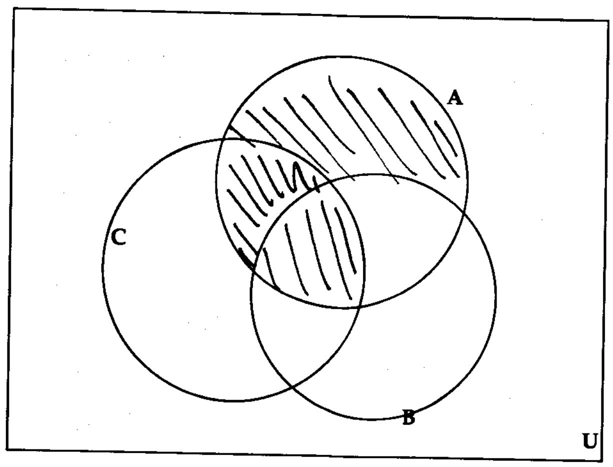
All students learn something.
All people enrolled in Math 112 are students.
Conclusion: All people enrolled in Math 112 learn something.

(d) $\sim(p \wedge q) \Leftrightarrow \sim p \wedge \sim q$ F

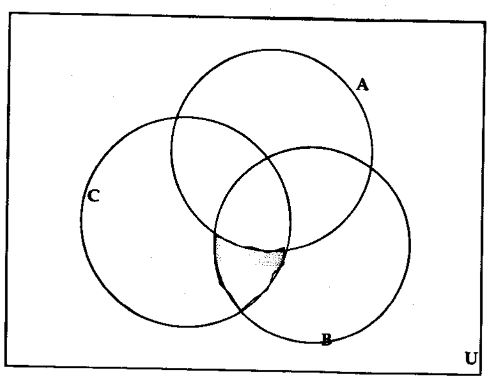
MATH 112 Practice 2 (for Exam 1)

Professor Goldin

1. *6 points* Consider the Venn diagram below. Shade in the region $A \cap (B' \cup C)$.



2. *6 points* Consider the following region. Describe it in terms of intersections and unions of A, B, C.



$A' \cap (B \cap C)$

3. *16 points* A campus survey asked 250 students what kind of pizza they like. The survey revealed that 125 like plain cheese pizza, 60 like pepperoni pizza, and 61 like olive pizza. Moreover, 37 like plain cheese and pepperoni pizza, 20 like pepperoni and olive pizza, and 8 liked all three kinds of pizza. There are 16 people who like plain and olive pizza.

(a) How many students like pepperoni but not plain pizza?

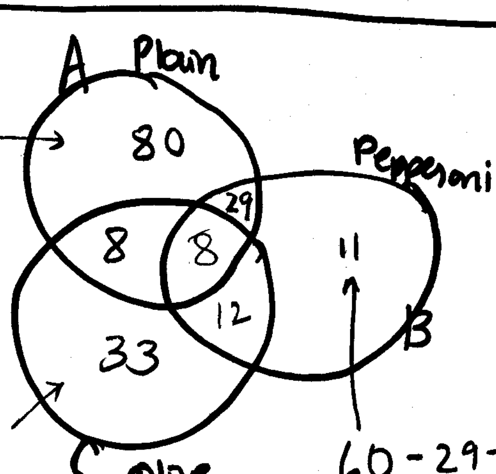
(b) How many students like pepperoni or olive, but not both?

(c) How many students like plain cheese only?

(d) How many students like plain cheese or olive, but not pepperoni?

$U = 250$

$125 - 29 - 8 - 8 = 80$



$61 - 8 - 8 - 12 = 33$

$60 - 29 - 12 - 8 = 11$

$|A| = 125$

$|B| = 60$

$|C| = 61$

$|A \cap B| = 37$

$|B \cap C| = 20$

$|A \cap B \cap C| = 8$

$|A \cap C| = 16$

(a) $|B \cap A'| = 60 - 29 = 31$

(or $8 + 12 + 11 = 31$)

(b) $B \cup C \cap (B \cap C)'$

$|B \cup C| = |B| + |C| - |B \cap C| = 60 + 61 - 20 = 101$

$101 - 20 = 81$

(c) $|A \cap B' \cap C'| = 80$

(d) $(A \cup C) \cap B'$

$= 8 + 33 + 80 = 131$

MAKESURE YOU KNOW PRINCIPLE OF INC/EXCL!

Sol'n not provided.

4. *6 points* A kid walks into an ice cream store and orders a double-scoop on a cone with one topping. There are 20 flavors of ice cream to choose from, and he may choose two scoops of the same flavor. He also chooses one of 4 types of cones, and one of 9 toppings. How many possible choices does he have? Keep in mind that a cone with chocolate on the bottom scoop and vanilla on the top scoop is different than a cone with chocolate on the top scoop, and vanilla on the bottom scoop.

5. *6 points* A company is looking to hire. It interviews 11 people, and intends to invite 5 of the candidates back for a second round of interviews. In how many ways can the company choose 5 candidates to invite back?

$$C(11,5) = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

6. *6 points* In creating a box of chocolates, Kerry had to put 16 distinct chocolates in a box. She was told to make the box so that it contains 6 dark chocolates, 5 white chocolates, and 5 milk chocolates. There were 8 different kinds of dark chocolates, 5 different kinds of white chocolate, and 10 different kinds of milk chocolate. All told, how many different ways could she make a box of chocolate?

$$C(8,6) \cdot C(5,5) \cdot C(10,5)$$

7. *6 points* In designing some stationary, Mark had to choose among 5 qualities of paper, 5 colors, 6 logos, and 4 colors for each logo. He had to choose one of each. How many ways could he make the stationary?

$$5 \cdot 5 \cdot 6 \cdot 4$$

8. *12 points* There are 110 people competing in a marathon. The first and second to finish get medals saying "1st" and "2nd", respectively. The next 8 to arrive at the finish line get a medal saying "top 10".

   (a) In how many ways could the 1st and 2nd place medals be won?

   ~~(b) In how many ways could the top-10 medals be won? *[You do not have to multiply out large number, but your answer should consist only use basic operations, not permutations and combinations.]*~~ UNCLEAR QUESTION

   (c) In total, how many ways could all 10 medals be handed out among the 110 runners? *[You do not have to multiply out large number, but your answer should consist only use basic operations, not permutations and combinations.]*

(a) $110 \cdot 109$

(b) $110 \cdot 109 \cdot C(108, 8)$

9. *12 points* An urn contains 31 distinctly numbered balls, of which 11 are red, 12 are blue and 8 are white. A sample of 6 balls is to be selected.

   (a) How many different samples are possible?
   (b) How many samples contain all red balls?
   (c) How many different samples contain exactly 5 blue and 1 white ball?
   (d) How many samples contain exactly 3 white balls?

(a) 31 balls, pick 6. $C(31, 6)$

(b) $C(11, 6)$ (pick 6 of 11 balls)

(c) $C(12, 5) \cdot C(8, 1)$

(d) $C(8, 3) \cdot$ ~~C([illegible])~~ $C(23, 3)$

non-white balls.

10. *6 points* Jack flips a coin 12 times.

(a) In how many ways could he get exactly 3 tails?

(b) In how many ways could he get at least 3 tails? (i.e. 3 tails or more)

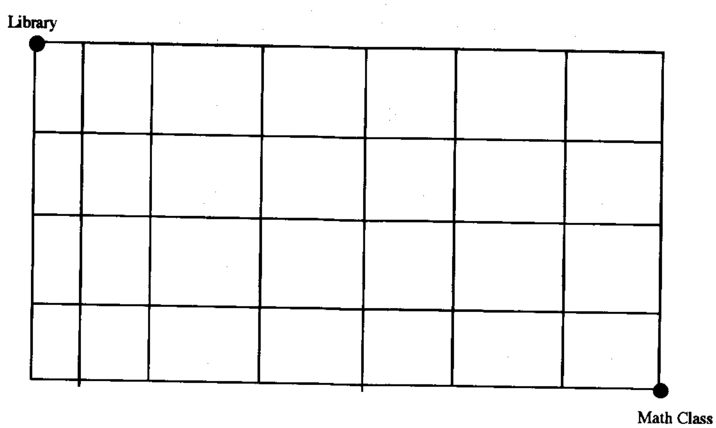
(a) $C(12,3) = \dfrac{12 \cdot 11 \cdot 10}{3 \cdot 2 \cdot 1}$

(b) $C(12,3) + C(12,4) + C(12,5) + C(12,6) + C(12,7)$
$+ C(12,8) + C(12,9) + C(12,10)$
$+ C(12,11) + C(12,12)$

-OR-

$2^{12} - (C(12,0) + C(12,1) + C(12,2))$

11. *6 points* A student plans to go from the library to point his math class on the map below. He can only go to the right or down, and he must move along the grid. How many ways can the student do this?



There are 4 Down moves and 7 Right moves.
To get from library to math, he must
move 4 D out of 11 moves. He can go D at any time.
$C(11,4)$ ways

12. *6 points* Consider the expansion of $(x+y)^{14}$ into monomials.

(a) What is the coefficient of x^3y^{11}?

(b) List another term with the *exact same* value of the coefficient.

(a) $C(14, 3)$

(b) $x^{11}y^3$

13. *6 points* Cathy's Catering offers fruit platters with 8 choices of fruit. She will make a fancy platter with any subset of these fruits you choose. How many possible fruit combinations could you order? Assume that you choose at least one fruit.

We are looking for nonempty subsets of a set of 8 fruits.

There are 2^8 subsets of a set of size 8, one of which is the emptyset.

So total possibilities are

$$2^8 - 1 = 255.$$