

12.3

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Give the hypothesis and conclusion
in each statement.

(a) Healthy people live a long life.

Hypothesis: Someone is a healthy person
 P (or: you are a healthy person).

Conclusion: This person will live
 q a long life

(or: you will ~~long~~ live a long life)

$$P \rightarrow q$$

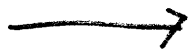
"If you're a healthy person, you'll live a long life."

(b) The ~~bus~~ ^{bus} stops requests it.

only if a person

↑
the condition "a person requests a stop" is required but not necessarily sufficient.

The bus stops



Someone requested a stop

p = the bus stops

q = ~~the~~ ^A person requests a stop.

$$p \rightarrow q$$

~~The~~ ^A wrong answer: $q \rightarrow p$
"If someone requests a stop,
the bus stops."

~~no~~ ~~one~~ ~~requests~~ \rightarrow it won't
a stop stop

$$\sim q \rightarrow \sim p$$

LOGICALLY EQUIVALENT

One is the contrapositive
of the other.

(c) For the azalea plant to grow, it is necessary for it to be exposed to sunlight.

q: the azalea plant grows

p: it is exposed to sunlight.
the azalea plant

If the azalea is not exposed to sunlight, it will not grow.

$$\sim p \rightarrow \sim q.$$

-or- contrapositive:

$$q \rightarrow p$$

"If the azalea plant grows, it is exposed to sunlight."

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$$(p \rightarrow q) \iff \sim(p \wedge \sim q)$$

R S

Using logical equivalence

		R		S	
p	q	$p \rightarrow q$	$\sim q$	$p \wedge \sim q$	$\sim(p \wedge \sim q)$
T	T	T	F	F	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	F	T

$R \iff S$

Using logical implication

$R \implies S$: Need to show $R \rightarrow S$ is a tautology.

R	S	$R \rightarrow S$
T	T	T
F	T	T
T	F	F
F	F	T

R	S	$R \rightarrow S$
T	T	T
F	F	T

NOT POSSIBLE

$R \rightarrow S$ is a tautology

No matter what p, q are,
R and S share a truth value.

$$\implies (R \implies S)$$

Logical Equiv. vs. Tautology

$$P \Leftrightarrow Q$$

means $P \Rightarrow Q$ (logical implication)

$P \rightarrow Q$ is a tautology (always true)

and

$$Q \Rightarrow P$$

$Q \rightarrow P$ is a tautology

Claim If $P \Rightarrow Q$ and $Q \Rightarrow P$, then
 $P \Leftrightarrow Q$ (logically equivalent)

We need to show $S \Rightarrow R$ (or $S \rightarrow R$ is a tautology)

R	S	R	$S \rightarrow R$
	T	T	T
	F	F	T

Thus $S \rightarrow R$ is a tautology

So $S \Rightarrow R$.

Yet another way:

$$\sim(p \wedge \sim q) \iff \sim p \vee (\sim \sim q)$$

$$\iff \sim p \vee q$$

$$p \rightarrow q \iff q \vee \sim p$$

These are equivalent by commutativity.

What is $\sim(p \rightarrow q)$

p	q	$p \rightarrow q$	$\sim(p \rightarrow q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

"If ^pyou work hard at maths, ^qyou can learn anything."

$\sim(p \rightarrow q)$

• Prop'n (A) "If you don't work hard, you won't learn anything."
 $\sim p \rightarrow \sim q$

(B) "If you don't work hard, ~~you~~ ~~can't learn every~~ there are some things you can't learn."

(C) "There are some things you can't learn if you don't work hard."

(D) "If you don't work ~~hard~~ hard, it doesn't mean you won't learn anything."

(E) "It is not the case that if you work hard you can learn anything."

Hard to distinguish b/c of "anything."

(F) You can learn anything even
if you don't work hard at math.