12.3
#43

Give the hypothesis and conclusion in each statement.

(a) Healthy people live a long life.

Hypothesis: Someone is a healthy person
p (or: you are a healthy person).

Conclusion: This person will ~~live~~ a long life
q (or: you will ~~live~~ a long life).

$p \rightarrow q$ "If you're a healthy person, you'll live a long life."

(b) The bus stops only if a person requests it.

The bus stops → someone requested a stop

the condition "a person requests a stop" is required but not necessarily sufficient.

p = the bus stops

q = A person requests a stop.

$p \rightarrow q$

A wrong answer: $q \rightarrow p$

"If someone requests a stop, the bus stops."

no one requests a stop $\longrightarrow$ it won't stop

$\sim q \longrightarrow \sim p$

LOGICALLY EQUIVALENT

One is the contrapositive of the other.

(c) For the azalea plant to grow, it is necessary for it to be exposed to sunlight.

q : the azalea plant grows

p : it (the azalea plant) is exposed to sunlight.

If the azalea is not exposed to sunlight, it will not grow.

$$\sim p \longrightarrow \sim q .$$

— or — contrapositive:

$$q \longrightarrow p$$

"~~If~~ ~~the~~ the azalea plant grows, it is exposed to sunlight."

3. p. 32

$(p \to q) \iff \sim(p \wedge \sim q)$

R S

Using logical equivalence

p	q	R: $p \to q$	$\sim q$	$p \wedge \sim q$	S: $\sim(p \wedge \sim q)$
T	T	T	F	F	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	F	T

$R \iff S$

Using logical implication

$R \Rightarrow S$. Need to show $R \to S$ is a tautology.

R	S	$R \to S$
T	T	
F	T	
T	F	
F	F	

NOT POSSIBLE

No matter what p, q are, R and S share a truth value.

$\Rightarrow (R \Rightarrow S)$

R	S	$R \to S$
T	T	T
F	F	T

$R \to S$ is a tautology

Logical Equiv. vs. Tautology

$P \Leftrightarrow Q$

means $P \Rightarrow Q$ (Logical implication)

$P \to Q$ is a tautology (always true)

and

$Q \Rightarrow P$

$Q \to P$ is a tautology

Claim If $P \Rightarrow Q$ and $Q \Rightarrow P$, then

$P \Leftrightarrow Q$ (logically equivalent)

We need to show $S \Rightarrow R$ (or $S \to R$ is a tautology)

R	S	R	$S \to R$
T	T	T	T
F	F	F	T

Thus $S \to R$ is a tautology

So $S \Rightarrow R$.

Yet another way:

$$\sim(p \wedge \sim q) \iff \sim p \vee (\sim\sim q)$$

$$\iff \sim p \vee q$$

$$p \to q \iff q \vee \sim p$$

These are equivalent by commutativity.

What is $\sim(p \to q)$

p	q	$p \to q$	$\sim(p \to q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

p

"If you work hard at math, you can learn anything."

q

$\sim(p \to q)$

- ~~Prop~~ (A) If you don't work hard, you won't learn anything.

$p \to \sim q$

(B) "If you don't work hard, ~~you can't learn every~~ there are some things you can't learn."

(C) "There are somethings you can't learn if you don't work hard."

(D) If you don't work hard, it doesn't mean you won't learn anything."

(E) It is not the case that if you work hard you can learn anything."

Hard to distinguish b/c of "anything."

(F) You can learn anything even if you don't work hard at math.