EXAM OCT 7 — Most likely to be Ch 12 & Ch 5 through 5.7

NOT 12.5 nor 5.8.

TO BE CONFIRMED WED.

Will post some practice test material → check back!

SIMPLE CALCULATOR : ONLY $\times, \div, +, -$

NOT NEEDED

§5.4

23. D 4 letter words h, o, t, s, m, x and e with the condition

(d) Words must end in a vowel, repetitions not allowed.

Problem with mult. principle:

If ~~these~~ op 1 = write down 1^{st} letter

op 2 = " " 2^{nd} letter

op 3 — — — 3^{rd} "

op 4 = — — — 4^{th} letter,

m_1 = # of ways to ~~do~~ carry out op. 1 $= 7$

$m_2 = 6$

$m_3 = 5$

m_4 is not independent of the outcomes for earlier operations.

Change the operations to complete the task!

op 1 : write down last letter $m_1 = 2$

op 2 : write down 3^{rd} letter $m_2 = 6$

op 3 : write down 2^{nd} letter $m_3 = 5$

op 4 : 1^{st} letter $m_4 = 4$

Ans $m_1 \cdot m_2 \cdot m_3 \cdot m_4 = 2 \cdot 6 \cdot 5 \cdot 4 = \underline{240}$

§5.4 Exam has
19 5 T/F questions. How many ways can the exam be completed?

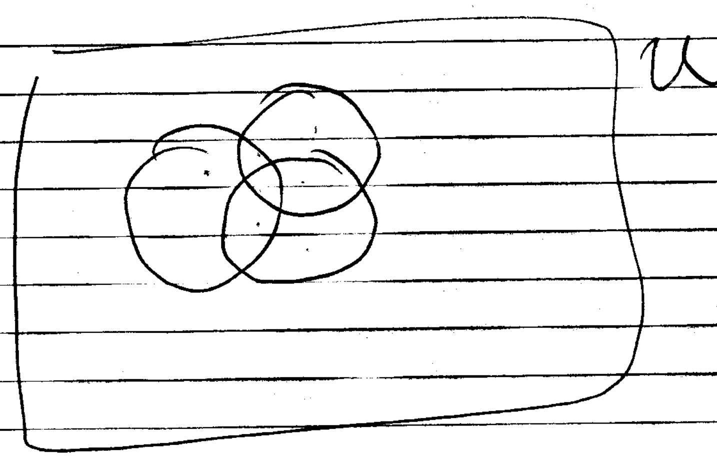
task: complete the test

op. 1 : Ans. Q 1 $m_1 = 2$
op. 2 Ans. Q2 $m_2 = 2$
Op 3 Q3 $m_3 = 2$
Op 4 Q4 $m_4 = 2$
Op 5 Q5 $m_5 = 2$

Ans : $2^5 = 32$

27. Shading: shaded or not.

Venn Diag. for 3 sets:



How many regions? 8 regions.
Each region is shaded or not.
8 operations of shading or not each region; for each of these $m_i = 2$
$i = 1, 2, \ldots, 8$

Ans $\underbrace{2 \cdot 2 \cdot 2 \cdots 2}_{8 \text{ times}} = 2^8 = \underline{256}$.

31. 6 breakfast, 7 lunch, 4 dinner } Choose one of each.

\# of ways to do this = # of days w/o repeating the menu.

op. 1 pick breakfast $m_1 = 6$
op 2 pick lunch $m_2 = 7$
op 3 pick dinner $m_3 = 4$

Mult. princ: Ans $m_1 \cdot m_2 \cdot m_3 = 6 \cdot 7 \cdot 4$

Last Class

If we have 100 books, and we want to put 5 on a shelf in order. How many ways to do this?

op 1 Put 1st book up $m_1 = 100$
op 2 2nd $m_2 = 99$
op 3 3rd $m_3 = 98$
op 4 4th $m_4 = 97$
op 5 5th $m_5 = 96$

Ans $100 \cdot 99 \cdot 98 \cdot 97 \cdot 96 = P(100, 5)$

100 items; choose 5

PERMUTATION
keep track of order!

Notation

$P(n, 1) = n$
$P(n, 2) = n \cdot (n-1)$
$P(n, 3) = n(n-1)(n-2)$
$\vdots$

$$P(n, k) = \underbrace{n(n-1)(n-2) \cdots (n-k+1)}_{k \text{ terms}}$$

= # of permutations of n objects taken k at a time

ORDER MATTERS.

Ex

Form a sequence of 6 cards from a deck of 52. How many ways can I do this?

(Note the order of the cards matters)

Ans : $52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47$

mult. principal

$= P(52, 6)$

Note $P(52,52) = 52 \cdot 51 \cdot 50 \cdot 49 \cdots\cdots 1$

$= 52!$

factorial

$n! = P(n,n) = n(n-1)(n-2)\cdots 1$

With permutations, order matters!

With combinations, order does not matter!

Ex Pick a pair (2) of puppies ~~for~~ out of a set of 20 puppies at the dog pound. How many ways can I do that?

Pick 1st puppy. There are 20 ways.
2nd 19

But if I first pick white spotted puppy
then brown fluffy puppy,
I have the same puppies as if I had picked them in a different order!
We counted two ways to get these puppies when really the result is the same pair.

$20 \cdot 19$ is twice as many possible pairs of puppies because each pair is counted twice.

The # of pairs of puppies $\frac{20 \cdot 19}{2}$.

General Principle

How many ways can I choose a set of k from n objects when the order doesn't matter?

Use algebra to solve this: Call this $C(n,k)$

(# of combinations of k objects from a set of n).

We want a formula for $C(n,k)$.

We do know a formula for $P(n,k)$

$$P(n,k) = \underbrace{n(n-1)\cdots(n-k+1)}_{k \text{ terms}}$$

← Picking k objects from n where order matters.

Break the task of choosing k objects from n in order into 2 operations

Op 1 Pick k objects from n without worrying about the order.

Op 2 Order the k objects I picked.

Ex Books: 100 books. Pick 6 to put on shelf in order.

$$\underbrace{100 \cdot 99 \cdot 98 \cdot 97 \cdot 96 \cdot 95}_{6 \text{ operations}}$$

$\underbrace{\text{Pick out 6 books} \quad \text{Op 1} \qquad \text{Put them in order} \quad \text{Op 2}}_{\text{2 operations}}$

P(n,k) = (# of ways to pick k objects from n without ordering them) (# of ways to order the k objects)

P(100, 6) = (# of ways to pick 6 books from 100) (# of ways to order the 6 books)

$\underbrace{P(100,6)}_{\text{\# of ways to put 6 books in order on shelf}} = \underbrace{(\text{\# of ways to pick 6 books from 100})}_{C(100,6)} \cdot 6!$

$100 \cdot 99 \cdot 98 \cdot 97 \cdot 96 \cdot 95 = C(100,6) \cdot (6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)$

$$\boxed{\frac{100 \cdot 99 \cdot 98 \cdot 97 \cdot 96 \cdot 95}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = C(100,6)}$$

$P(n,k) = C(n,k) \cdot k(k-1)(k-2)\cdots 1$

$n(n-1)\cdots(n-k+1) = C(n,k) \cdot k(k-1)(k-2)\cdots 1$

$$\Rightarrow \frac{n(n-1)(n-2)\cdots(n-k+1)}{k(k-1)(k-2)\cdots 1} = \underbrace{C(n,k)}_{\text{\# of ways to choose k objects from n without order.}}$$

Ex, Pick 2 puppies from 20:

$$C(20, 2) = \frac{20 \cdot 19}{2 \cdot 1}$$

$n = 20$
$k = 2$

Calculate $C(5,3)$ $\quad n=5$, $k=3$

$$= \frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} = 10$$

When to use one or other?

There are 10 students, and I take a pick of 4 sitting in a row of 4 chairs.

How many ways can I do this?
order matters
Ans. $P(10,4) = 10 \cdot 9 \cdot 8 \cdot 7$

I have a contest with 50 people competing for 1st 2nd & 3rd prizes. How many different outcomes are there?

order matters Ans $P(50,3)$
$= 50 \cdot 49 \cdot 48.$

I walk into a fruit store, with 10 varieties of fruit. The 3-fruit Special says I can fill my plate with 3 fruit varieties. How many ways can I fill it?

order doesn't matter $C(10,3)$

$$= \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1}$$ Combinations

Flip a coin 10 times.
How many outcomes have
exactly 4 heads & 6 tails?

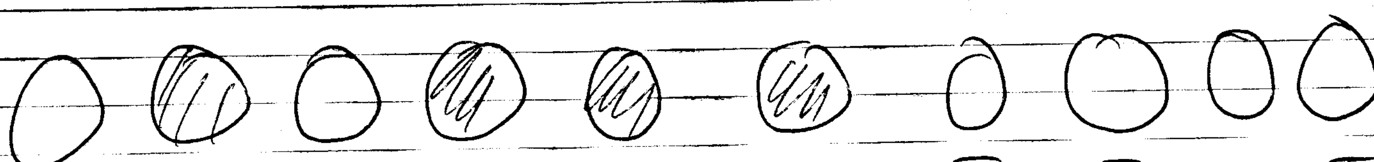
2^{10} outcomes for 10 flips in a row.
$= 1024$

<u>Trick</u> Think about which flips are H.
Among the 10 flips, <u>4 must be H</u>
(and the rest tails)

For each flip of 10 coins with 4 H, I have
a choice of 4 coins that are H.

For each choice of 4 coins that are H,
I have a sequence of 10 flips
with exactly 4 H.

The # of 10 flips with exactly 4 H
is the # of choices of 4 coins from 10.



T H T H H H T T T T

<u>Combinations</u> $C(10,4)$ = # ways to pick 4 coins from 10.