Office HRS today CANCELLED
but see me after class for any brief questions

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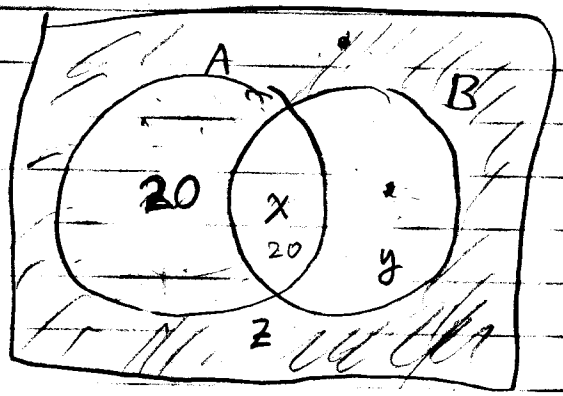

U = 100 workers
A = college graduates
B = union members

60 not college grads
20 non union college grads
30 union members
How many were neither college grads nor union members?

$|A'| = 60$ $B' \cap A$ = {non union college grads}

$|B' \cap A| = 20$

$|B| = 30$ Want $|A' \cap B'| = |(A \cup B)'|$

↑ deMorgan

Technique: introduce a variable or two (or three)

note $x = |A \cap B|$

$y = |B \cap A'|$

$z = |(A \cup B)'|$ ← This is what we want!

$|U| = |A| + |A'|$

$100 = |A| + 60 \Rightarrow |A| = 40$.

$|A| = 20 + x \Rightarrow 40 = 20 + x \Rightarrow x = 20$

$U = |B| + |B'| \Rightarrow 100 = 30 + |B'| \Rightarrow |B'| = 70$

$|B'| = 20 + z \Rightarrow 70 = 20 + z \Rightarrow \boxed{z = 50}$

To figure out y: $|B| = x + y = 20 + y \Rightarrow y = 10$

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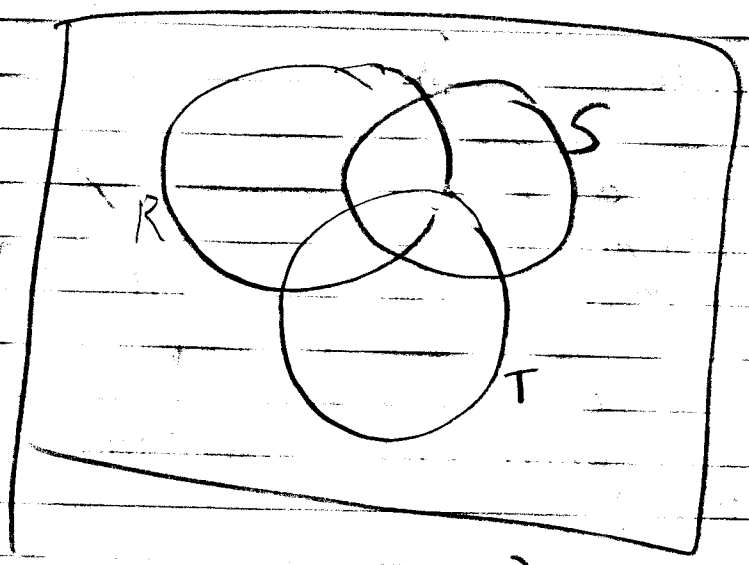
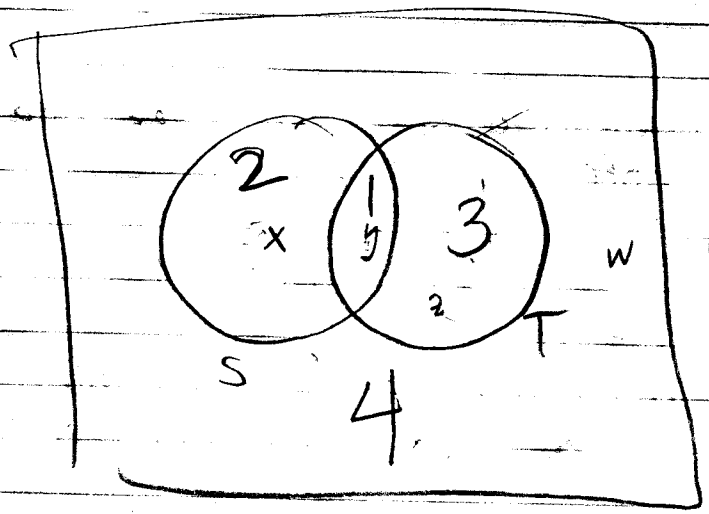


$n(S) = 3$ $n(S \cup T) = 6$, $n(T) = 4$

$n(S' \cup T') = 9$

R not relevant



$x + y = 3$

$x + y + z = 6$

$y + z = 4$

$w + x + z = 9$

$x + z + 3 = 9$

$z = 3$

$S' \cup T' = (S \cap T)'$

DEMORGAN

Always using PRINCIPLE OF INCLUSION + EXCLUSION

$|S \cup T| = |S| + |T| - |S \cap T|$

$6 = 3 + 4 - |S \cap T| \Rightarrow |S \cap T| = 1$

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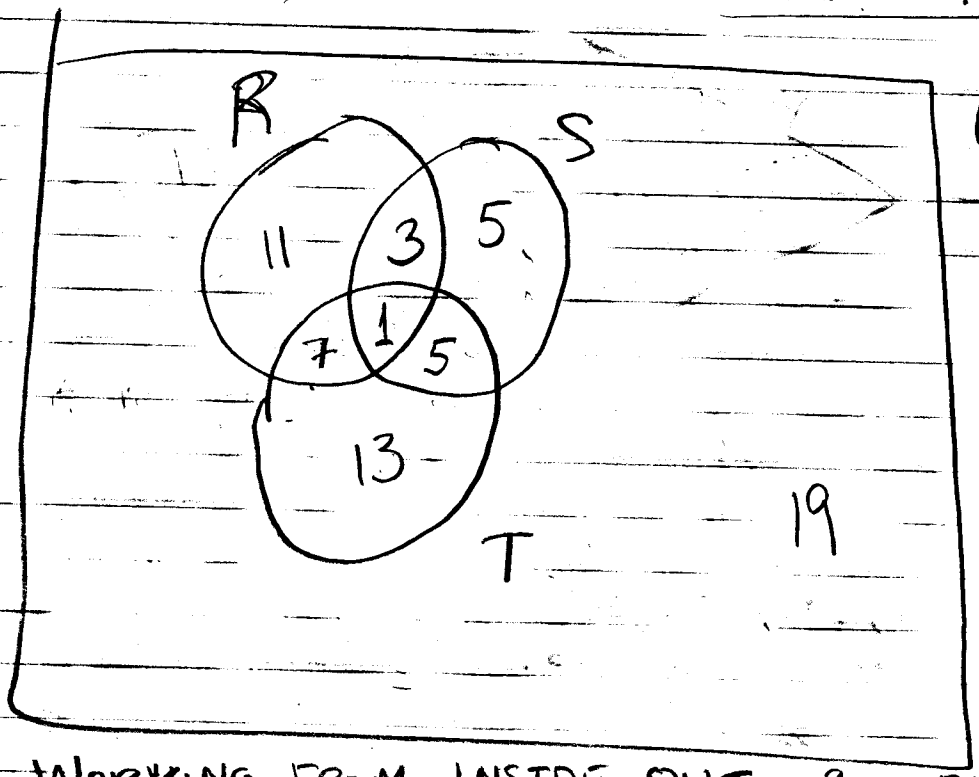
$n(U) = 64$ $n(R \cup S \cup T) = 45$ $n(R) = 22$

$n(T) = 26$ $n(R \cap S) = 4$ $n(S \cap T) = 6$

$n(R \cap T) = 8$ $n(R \cap S \cap T) = 1$



WORKING FROM INSIDE OUT BY REGION.

$|T| = 26 = 7 + 1 + 5 + ?$ 13 :)

$|R| = 22 = 7 + 3 + 1 + ?$ 11

$n(R \cup S \cup T) = 45 = 11 + 3 + 7 + 1 + 5 + 13 + ?$

$= 40 + ?$ 5

$|U| = |R \cup S \cup T| + ?$

$64 = 45 + ?$ 19

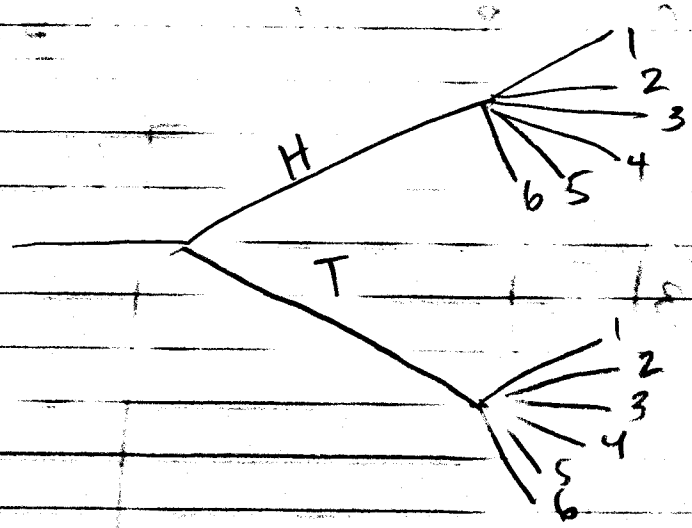
Using PIE

$n(R \cup S \cup T) = n(R) + n(S) + n(T) - n(R \cap S) - n(R \cap T) - n(S \cap T) + n(R \cap S \cap T)$.

$\Rightarrow 45 = 22 + n(S) + 26 - 4 - 8 - 6 + 1$

$45 = 31 + n(S) \Rightarrow n(S) = 14$ Get rest by doing more PIE.

Counting exercise: I flip a coin, then I roll a 6-sided die, how many outcomes are there? possible



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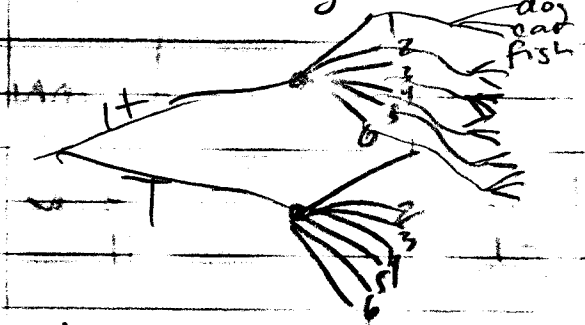
flip a coin, roll a 6-sided die, then pick one of 3 dog, cat, fish how many outcomes.



$2 \times 6 \times 3 = 36$

MULTIPLICATION PRINCIPLE.

If a task is composed of two operations, and the first operation has m outcomes, and for each of the first outcomes, there are n outcomes for the second operation, then there are $m \cdot n$ outcomes of the task.

Does this violate the mult. principle?

Task: flip a coin. If heads, pick a card from all the red cards in a standard deck of 52 cards.
If tails, pick a card from all the black cards in a standard deck.

General mult. principle: Suppose a task consists of t operations performed consecutively.
Suppose operation 1 can be performed in m_1 ways, and for each of these, operation 2 can be performed in m_2 ways, and for each of these, operation 3 can be performed in m_3 ways, etc,

then the total # of ways to perform the task is
$m_1 \cdot m_2 \cdot m_3 \cdot \cdots \cdot m_t$.

Example

I want to assign social security numbers to people working in the U.S.
These are 9 digits long.
How many ways to do this?

The task is a sequence of 9 operations of picking a # from 0 to 9.

Operation 1 : 10 ways $m_1 = 10$
2 : 10 ways $m_2 = 10$
3
:
$m_i = 10$

By mult. principle, there are
$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$
$= 10^9$ different #s.

Special Case of Mult Princ. is PERMUTATIONS

Ex. I have 4 books (Math, Sci Fi, Novel, Self help)
How many ways can I arrange them on the shelf?

Operation 1: put 1st book on shelf $m_1 = 4$
2: put 2nd book to right of 1st book $m_2 = 3$
3: put 3rd book to right of 2nd $m_3 = 2$
4: put last book to right of 3rd $m_4 = 1$

Ans. by mult. prin. is $4 \cdot 3 \cdot 2 \cdot 1 = 4!$
↑ factorial.

Ex Design a test with 20 problems in order from a test bank with 100 problems.
How many ways can I write the test?
Operation i is pick the i^{th} problem for the test $(i = 1, 2, \ldots, 20)$.

$m_1 = 100$
$m_2 = 99$
$m_3 = 98$
$\vdots$
$m_{20} = 81$

Ans $100 \cdot 99 \cdot 98 \cdots 81$