

Recall \forall for all
 \exists there exists

Last time:

$$\sim (\exists x p(x)) \Leftrightarrow \forall x \sim p(x)$$

$$\sim (\forall x p(x)) \Leftrightarrow \exists x \sim p(x).$$

Predicates with two variables:

x, y two variable don't need to be in same universe, but they may be.

~~$p(x) = x$ is rich~~

~~$U = \text{GMU students}$~~

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$p(x, y) = "x \text{ is richer than } y."$

$q(x, y) = "x \text{ is a better student than } y."$

$$\exists x, y \in U (p(x, y) \wedge q(x, y))$$

"There ~~exists~~ ^{are} students x and y such that x is richer and a better student than y ."

MUST BE
SAME STUDENTS
FOR BOTH OPEN SENTENCES

Result of full
 E then
 last time

$$(x \rightarrow y) \Leftrightarrow (x \wedge \neg y) \rightarrow \perp$$

$$(x \rightarrow y) \Leftrightarrow (\neg x \vee y)$$

Propositions with two variables
 if two variables then they can be
 combined into one variable

" $x \rightarrow y$ " is a tautology
 "not a tautology"
 "not a tautology"
 "not a tautology"

$$(x \rightarrow y) \wedge (y \rightarrow x) \Leftrightarrow (x \leftrightarrow y)$$

both sides of the equation
 are true
 for both sides
 true
 true
 true

$$\forall y \forall x [3x - 5 = y]$$

For any real #s x and y , $3x - 5 = y$

FALSE

14 a. For all stitches x , x saves time.

Negate: \exists stitches x s.t. x does not save time.

Answer: Some stitches do not save time.

c) \exists a child x ~~s.t.~~ ^{such that} x is afraid of snakes.

Negate:

\forall children x , x is not afraid of snakes.

$\sim p(x)$

English: No children are afraid of snakes. OR

$\left\{ \begin{array}{l} \text{All children are unafraid} \\ \text{of snakes} \end{array} \right.$

BEST ANSWER!

$U = \text{GMU students.}$

$p(x, y) = "x \text{ has more siblings than } y."$

$\forall x \exists y p(x, y) = " \text{For all } \text{GMU} \text{ students } x$
 ~~$x \text{ has more siblings than student } y.$~~
there exists a student x
that has more siblings than
student $y."$

For all - or -
For every

For every GMU student x , there
① exists a student y such that
 x has more siblings than y .

② For any given GMU student x , there
exists a student y who has more
siblings than x .

These are not the same!

For every GMU student

Rewrite

① For every GMU student, there
exists another student with fewer
siblings.

$\exists x \forall y p(x,y)$

$p(x,y) =$ "x has equal or more siblings than y"

There exists a student who has ^{equal or more} siblings than anyone at GMU.

(This student has the max^{# of} siblings)

Negation of predicates in 2 variables

$\sim (\forall x \forall y p(x,y)) \Leftrightarrow \exists x \exists y (\sim p(x,y))$

$\sim (\exists x \forall y p(x,y)) \Leftrightarrow \forall x \exists y (\sim p(x,y))$

See p. 43 for more!

Example $U = \mathbb{Z}$ integers

$\forall x \exists y$ such that x divides y.

For every number x, there's a number y such that x divides it.

TRUE.

Example $U = \mathbb{R}$ real numbers

$\forall y \exists x [3x - 5 = y]$

For every # y, there is a choice of x such that $3x - 5 = y$. (Solve for x)

TRUE.

$$11. a \quad \forall x [p(x) \vee q(x)]$$

$$U = \{0, 1, 2, 3, \dots\} \quad \text{TRUE}$$

$p(x) \vee q(x) =$ "x is even or x is odd."

$$c) \exists x [p(x) \vee q(x)] \quad \text{TRUE}$$

$$b) \underbrace{[\forall x p(x)]}_{\text{T or F?}} \text{ or } \underbrace{[\forall x q(x)]}_F \quad \text{FALSE.}$$

F

$$10. \{3, 4, 5, 6, \dots\} = U$$

$$p(x) = [x \text{ is prime} \rightarrow x^2 + 1 \text{ is even}]$$

$$(a) \exists x p(x)$$

$p(3)$ TRUE, so $\exists x p(x)$ TRUE.

$$(b) \forall x p(x) \quad \text{TRUE}$$

$p(3)$ TRUE ✓

$p(4)$ ~~TRUE~~ (4 is not prime, so the implication is TRUE!)

$p(5)$ TRUE

$p(6)$ TRUE x not prime implies p(x)

⋮

x is prime implies TRUE.
TRUE.