Negating $p \to q$

$\sim(p \to q)$ What does this mean?

"If $\underbrace{3 \text{ is odd}}_{p}$, then $\underbrace{5^2 = 26}_{q}$."

$$p \to q$$

$\sim(p \to q)$ How do you say this in words?

or

Recall $p \to q \iff \underbrace{\sim q \to \sim p}_{\text{contrapositive}}$

$\iff \cancel{\sim q \vee p}$? NO

$\sim p \vee q$

$$\boxed{p \to q \iff \sim p \vee q}$$

$$\sim(p \to q) \iff \sim(\sim p \vee q)$$
$$\iff p \wedge \sim q$$

$\boxed{\text{"3 is odd and } 5^2 \neq 26\text{."}}$

"The person x is nice"

<u>Open sentence</u>. It becomes a statement when we specify x.

"The person John is nice"
"The person Marina is nice"
etc.

Need to specify a <u>universe</u> of values that x can take on.
For example, $U = \{$people in this classroom$\}$.

$p(x) =$ "The integer x is a square number."

If $U = \{1, 2, 3, 4, 5, \dots\}$
then $p(2)$ is false
$p(1)$ is true
$p(5)$ is false
$p(25)$ is true ....

"$p(x)$ is true for every element $x \in U$"
FALSE!
"$p(x)$ is true for some element $x \in U$" TRUE!

For every element x in $\mathcal{U}$	$\forall x \in \mathcal{U}$
There exists an element $x \in \mathcal{U}$ such that...	$\exists x \in \mathcal{U}$

<u>Ex</u> $\mathcal{U} = \{1, 2, 3, 4, \ldots\}$

$$\exists x \in \mathcal{U}, \; x \geq 5.$$

"There exists an elt x of $\mathcal{U}$ such that $x \geq 5$."

This doesn't specify x, but it <u>is</u> a statement (in this case, it's true.)

$$\forall x \in \mathcal{U}, \; x = 5.$$

"For all elements x in $\mathcal{U}$, $x = 5$."

FALSE.

<u>Example</u> $\forall x \in \mathcal{U} \quad (x-3)(x+4) > 0$

FALSE.

(when $x = 3$, $(x-3)(x+4) \not> 0$)

$\rightarrow \; \exists x \in \mathcal{U} \quad (x-3)(x+4) \not> 0.$

This is true.

$U = \{1, 2, 3, \ldots\}$

<u>Example</u>

$\exists x \in U \quad x^2 = 1$ TRUE

$\exists x \in U \quad x^2 = -1$ FALSE

Universe matters!

$\exists x \in U \quad x < 0$ FALSE

proved it's false by showing
$\forall x \in U, \quad x \not< 0$. TRUE

Negate the statement [The universe must be specified / understood]
$\underbrace{\forall x \; p(x)}_{\text{whole thing is statement}}$. $\left(\forall x \in U \; p(x) \right)$

(p(x) is not a statement).

$$\boxed{\begin{array}{c} \sim\left(\forall x \in U, \; p(x)\right) \Leftrightarrow \exists x \in U, \sim p(x). \\ \sim\left(\exists x \in U, p(x)\right) \Leftrightarrow \forall x \in U, \sim p(x). \\ \text{De Morgan's Laws} \end{array}}$$

$\forall$, $\exists$ are "predicates."

~~For all G~~

"GMU students are broke and hungry."

$U = \{\text{GMU students}\}$

$p(x) =$ "x is broke."

$q(x) =$ "x is hungry"

$$\forall x \in U,\ p(x) \wedge q(x).$$

Negate the statement:

"Some GMU students are not broke or not hungry."

$$\exists x \in U \sim\big(p(x) \wedge q(x)\big)$$

$$\Leftrightarrow \quad \exists x \in U \sim p(x) \vee \sim q(x)$$

MORE VARIABLES!

Let $U = \{\text{GMU students}\}$

$p(x,y) =$ "x has higher cholesterol ~~broker~~ than y."

$q(x,y) =$ "x has a nicer car than y."

What, in English, does this mean: 6

$$\forall x \in U \ \exists y \in U \ [p(x,y) \longrightarrow q(x,y)]$$

$p(x,y) \to q(x,y)$ "If x has higher chol. than y, then x has a nicer car than y."

For every GMU student x, there is a GMU student y such [if x has higher cholesterol, then x has a nicer car."]

Pick any student A

There is a student B s.t.

cholesteral (A) > chol (B)

$\longrightarrow$ car (A) nicer than car (B)

Suppose we pick A, and I find

a student B ~~st~~ whose chol.

is higher than A.

Then we've satisfied the condition.