Recall

$$Pr(E \mid F) = \frac{Pr(E \cap F)}{Pr(F)}.$$

provided $Pr(F) \neq 0$.

What is independence?

You're flipping coins, 1^{st} 8 are H.

Flip again, what's the prob. the next is H? $\frac{1}{2}$.

If E = event the 9^{th} flip is H.

F = event the 1^{st} 8 flips are H

$$Pr(E \mid F) = \frac{1}{2} = Pr(E)$$

"independent of ~~what~~ whether F occurred"

Flip 9 coins in a row

$S = \{$ sequences of H's and T's of length 9 $\}$ $n(S) = 2^9$

$n(E) = 2^8$ $E = \{$ ~~the~~ sequences of H's & T's with the 9^{th} entry H $\}$

$n(F) = 2$ $F = \{$ sequences of 9 H's & T's with first 8 ~~flip~~ entries are H $\}$

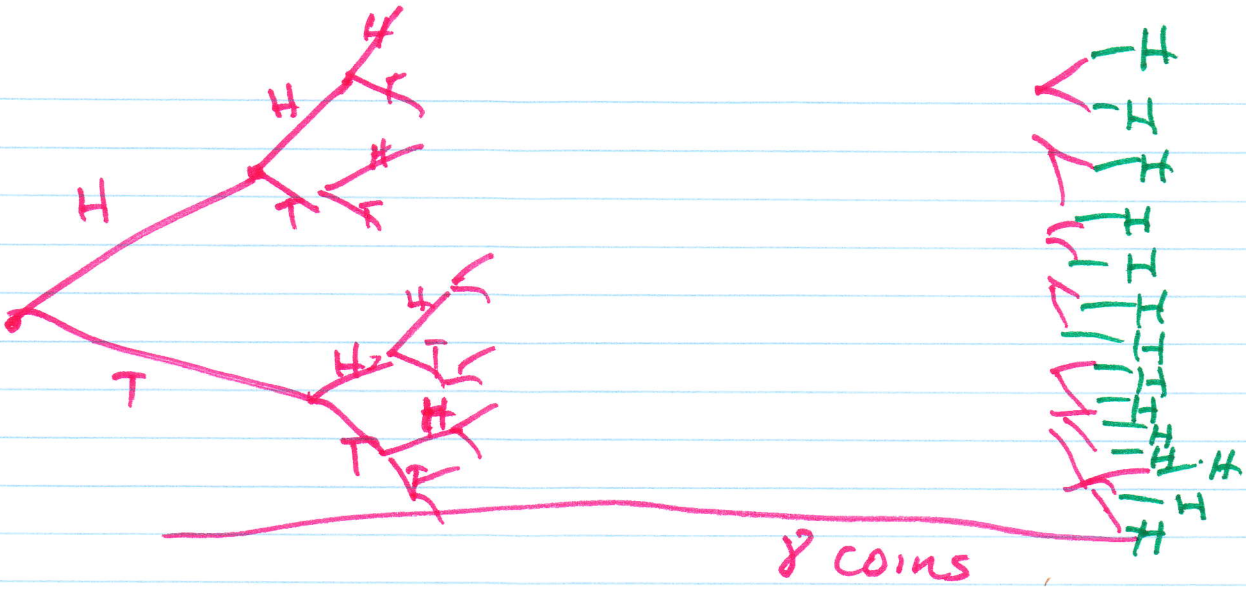
Let's see if $Pr(E \mid F) = Pr(E)$.

$$Pr(E) = \frac{n(E)}{n(S)} = \frac{2^8}{2^9} = \boxed{\frac{1}{2}}$$

$$E \cap F = \{ \{ \underbrace{HHHHHHHH}_{\text{because the outcome is in } F} H \} $$

$n(E \cap F) = 1$

because the outcome is in E



$$2^8$$

$$2^8 + 2^8 \qquad = 2(2^8) = 2^9$$

\# ways to finish with H

\# of ways to finish with T

$$Pr(E \mid F) = \frac{Pr(E \cap F)}{Pr(F)}$$

$$F = \left\{ \begin{array}{l} (H, H, H, H, H, H, H, H, H) \\ (H\ H\ H\ H\ H\ H\ H\ H, T) \end{array} \right\}$$

$\cancel{n(2)}$ $n(F) = 2$

$$Pr(F) = \frac{n(F)}{n(S)} = \frac{2}{2^9} = \frac{1}{2^8}$$

$$Pr(E \cap F) = ? = \frac{n(E \cap F)}{n(S)} = \frac{1}{2^9}$$

$$Pr(E \mid F) = \frac{1/2^9}{2/2^9} = \boxed{\frac{1}{2}}$$

Notice that $Pr(E \mid F) = Pr(E)$.

~~we say~~ This happens when E and F are independent events.

Def We say E and F are independent if $Pr(E \cap F) = Pr(E) \cdot Pr(F)$

In our case $\frac{1}{2^9} = \frac{1}{2} \cdot \frac{1}{2^8}$ ✓

Example Roll a 6 sided die, flip a coin, what's the prob. of getting 2 H.

Method 1 $S = \{ (1,H), (2,H), (3,H), (4,H), (5,H), (6,H), (1,T), (2,T), 3T, 4T, 5T, 6T \}$

$n(S) = 12$

$n(E) = 1$ $E = \{2H\}$

Ans. $Pr(E) = \frac{n(E)}{n(S)} = \frac{1}{12}$

Method 2 the coin flip is independent of the die roll.

Let E = event you get a 2 on die
F = event you get a H on coin.

$Pr(E \cap F) = Pr(2H) = \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}$.

More on independence:

4 children family:

E = event "at least one ~~boy~~ boy."

F = event "at least* one child of each sex."

(slightly different from text).

Are these events <u>independent</u>?

If they are, then $Pr(E \cap F) = Pr(E) \cdot Pr(F)$.

We simplify verify if this equation holds or not.

$E \cap F = \{$ BGBB GBBB BBGB BBBG
BGBB GGGB BGGG GBGG
⋮ $\}$

2^4 elements of S $= 16$

$n(E \cap F) = 14$
$n(E) = 15$
$n(F) = 14$

$Pr(E \cap F) = \frac{14}{16}$

$Pr(E) = \frac{15}{16}$

$Pr(F) = \frac{14}{16}$

Note $\frac{14}{16} \neq \frac{15}{16} \cdot \frac{14}{16}$ so they are not independent.

$Pr(E | F) = 1$

$Pr(F | E) = \frac{Pr(E \cap F)}{Pr(E)} = \frac{14/16}{15/16} = \underline{\frac{14}{15}}$

$$Pr(E|F) = \frac{Pr(E \cap F)}{Pr(F)}$$

$$\Rightarrow \boxed{Pr(E \cap F) = Pr(F)\, Pr(E|F)}$$ Product Rule

Generalized

$$Pr(E_1 \cap E_2 \cap E_3) = Pr(E_1)\, Pr(E_2|E_1) \cdot Pr(E_3|E_1 \cap E_2)$$

$$= Pr(E_1)\, Pr(E_2|E_1)\, Pr(E_3|E_2 \cap E_1)$$

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E_1 = 1st test correctly identifies blood type $\quad Pr(E_1) = .7$

E_2 = 2nd test correctly identifies type

$$Pr(E_2) = .8$$

at least one correctly identifies it:

$$Pr(E_1 \cup E_2) = .9$$

(a) Wants $Pr(E_1 \cap E_2)$

PIE $\quad Pr(E_1 \cup E_2) = Pr(E_1) + Pr(E_2) - Pr(E_1 \cap E_2)$

$$.9 = .7 + .8 - Pr(E_1 \cap E_2)$$

$\Rightarrow Pr(E_1 \cap E_2) = .6$ $\qquad$ solve

(b) $Pr(E_2 | E_1) = \dfrac{Pr(E_1 \cap E_2)}{Pr(E_1)}$

$= \dfrac{.6}{.7}$

(c)

(d) Are E_1 and E_2 independent?

$Pr(E_1 \cap E_2) \stackrel{?}{=} Pr(E_1)\, Pr(E_2)$

<u>No</u>! $\quad .6 \neq .7 \cdot .8 = .56$

(Also notice from (b) : if independent,

$Pr(E_2 | E_1) = Pr(E_2)$

$\overset{\shortparallel}{.6/.7} \neq \overset{\shortparallel}{.8}$ so not independent.