Conditional Probability

Oct 23

Conditions/ Information can change the probability of an event.

Roll 2 dice & record their face values.
$T = \{$ all ordered pairs of $1, \dots, 6\}$ $|T| = 36$

What's the probability you get $(6,6)$?

Ans $\frac{1}{36}$

What's the prob. you get $(6,6)$
conditional → if you know both dice got even # ?

Sample space shouldn't be all pairs, but just pairs of even #s.

$$S = \left\{ \begin{array}{ccc} (2,2), & (2,4), & (2,6) \\ (4,2) & (4,4) & (4,6) \\ (6,2) & (6,6) & (6,4) \end{array} \right\}$$

E

Ans $\frac{1}{9} = \frac{n(E)}{n(S)}$

Notice Roll 2 dice, what's the prob. of getting even #s on both.

$$\frac{9}{36} = \boxed{\frac{1}{4}}$$

And $\frac{1}{9} \cdot \frac{1}{4} = \frac{1}{36}$ Is there something going on?

$$\frac{1}{9} = \frac{1/36}{1/4}$$

Venn diagram

$E = \{(6,6)\}$

$S = \{$ even pairs $\}$

$$\underbrace{Pr(E \text{ given } S)}_{1/9} = \frac{Pr(E)}{Pr(S)} \quad \frac{1/36}{1/4}$$

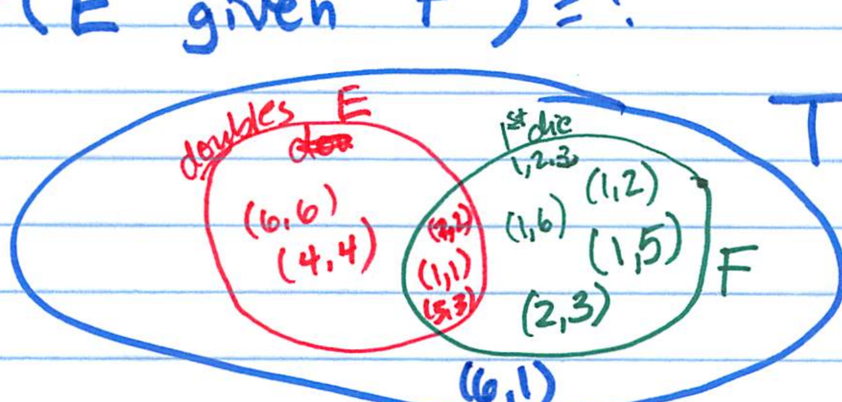
Slightly harder example. Roll 2 dice, record their face value.

$T = \{$ all ordered pairs of face values $\}$ $|T| = 36$

$E = \{$ ~~ordered pairs where 1st die~~ doubles $\}$

$F = \{$ ordered pairs first die is 1, 2, or 3 $\}$.

$Pr(E \text{ given } F) = ?$



Not all are written

$Pr(E \text{ given } F) = ?$

Pretend sample space is F

event is $E \cap F$

$$Pr(E \text{ given } F) = \frac{n(E \cap F)}{n(F)}$$

# of all outcomes of interest

# of all possible outcomes

How many rolls of 2 dice have 1, 2, or 3 on first die? 18 (mult. principle)

$n(F)$

How many rolls are doubles E among those with F 1, 2, 3 on 1st die? 3

$n(E \cap F)$

Ans $\frac{3}{18} = \frac{1}{6}$

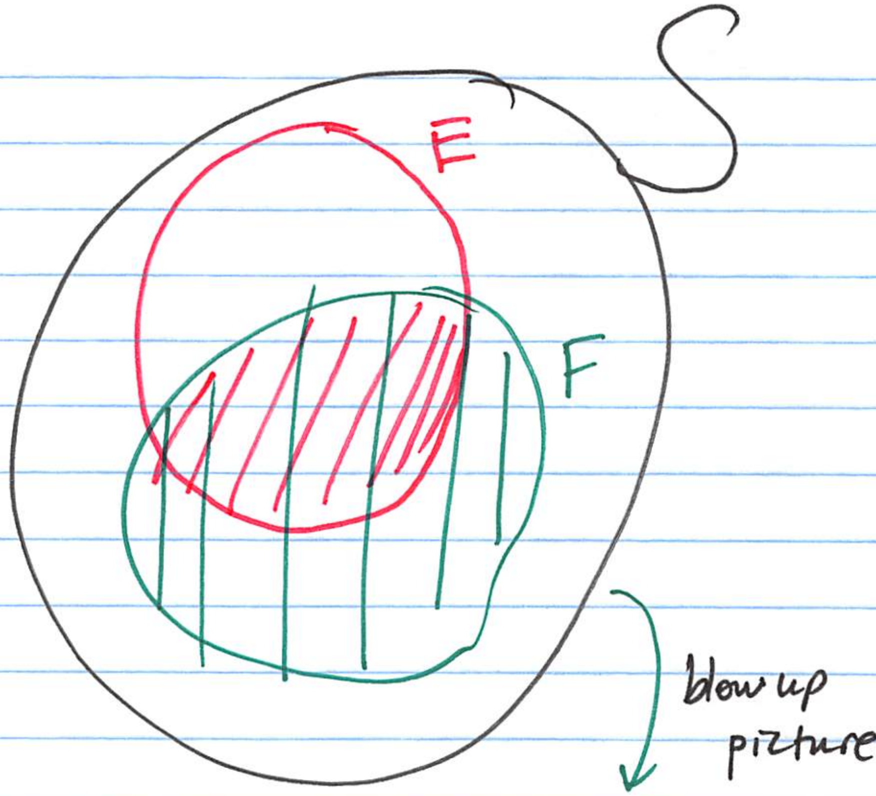
$$\frac{n(E \cap F)}{n(F)} = \frac{3}{18} = \frac{1}{6}.$$

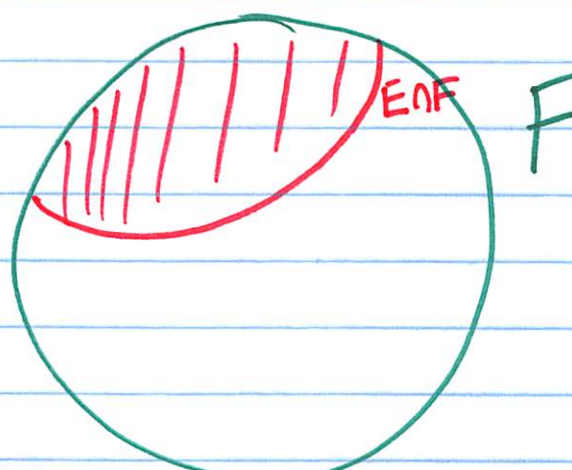
We write $Pr(E \mid F)$ to mean "the probability of E given F."

You assume F holds.

(or that you ~~are in~~ the event F happened).

$Pr(E|F)$





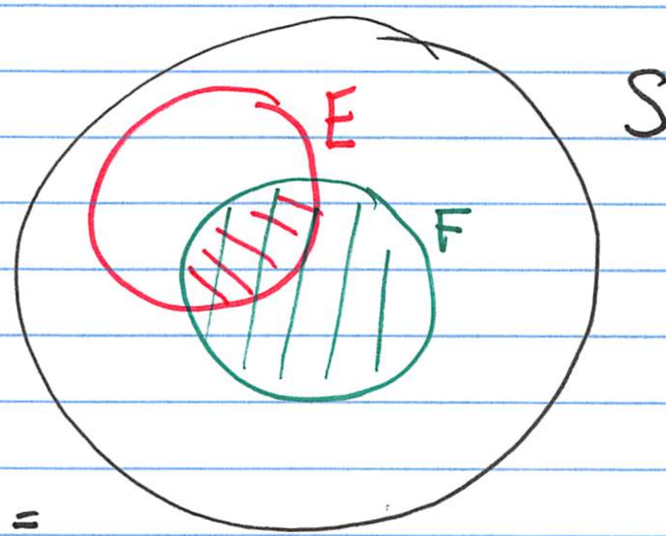
If every outcome in F is equally likely, then

$$Pr(E|F) = \frac{n(E\cap F)}{n(F)}$$

$\leftarrow$ $E\cap F$ is the event E occurring given that F occurs

$\uparrow$ F is the sample space we should use

If all outcomes are not equally likely....



Def $Pr(E|F) = \dfrac{Pr(E \cap F)}{Pr(F)}$ $\quad Pr(F) \neq 0$

If all outcomes were equally likely,

$$Pr(E \cap F) = \frac{n(E \cap F)}{n(S)}$$

$$Pr(F) = \frac{n(F)}{n(S)}$$

$$Pr(E|F) = \frac{\boxed{Pr(E \cap F)}}{Pr(F)} = \frac{\boxed{\dfrac{n(E \cap F)}{\cancel{n(S)}}}}{\dfrac{n(F)}{\cancel{n(S)}}} = \frac{n(E \cap F)}{n(F)}$$

which is what we said before.

Car towed	Probability
black Acura	.5
black Mercedes	.05
Silver Ultima	.1
Green Maxima	.06
tan Corolla	.04
blue Audi	.2
Silver BMW	.03
Red Volvo	.02

$$\Pr(\underbrace{\text{Silver car towed}}_{E} \mid \underbrace{\text{car was German}}_{F})$$

given

$\Pr(E \mid F)$

Assume car was German. How likely was it to be silver?

What proportion of the German cars were silver?

$$\frac{.03}{.03 + .2 + .05}$$

$E \cap F$ = silver & German

$$\Pr(E \mid F) = \frac{\Pr(E \cap F)}{\Pr(F)} = \frac{.03}{.03 + .2 + .05} = \frac{.03}{.28}$$

Multiply both sides by $Pr(F)$

$$Pr(E|F) \cdot Pr(F) = Pr(E \cap F)$$

"Multiplication Rule"

Ex At GMU, half of students are male and half are female.

80% of ~~students~~ ~~get of~~ women get financial aid and 70% of men.

What's the prob. that a randomly selected student will be a female with financial aid?

Let F = event a person is female

E = event a person is on financial aid.

$E \cap F$ = event person is female on FA

Want

$Pr(E \cap F)$

We know: $Pr(E \cap F) = Pr(E|F)Pr(F)$

$Pr(E|F) = .8$

$Pr(F) = .5$

$Pr(E \cap F) = .8 \cdot .5 = .40$ or 40%

§6.4

1. Each of 3 people randomly chooses one of 3 calculus sections to take (A, B, or C).

a) Prob all choose same one.

b) Prob all choose different sections.

$S = \{\text{ordered triples of } A, B \text{ or } C\}$ (A, A, A) (A, A, B) (A, A, C) ⋮

$n(S) = 27 = 3\cdot 3\cdot 3$

a) $E = \{(A,A,A) \ (B,B,B) \ (C,C,C)\}$

$Pr(E) = \dfrac{n(E)}{n(S)} = \dfrac{3}{27} = \dfrac{1}{9}$

b) $E = \{\text{all } \cancel{\text{separate}} \text{ different}\}$

$n(E) = 3!$

$(\cdot , \cdot , \ )$

$3 \cdot 2 \cdot 1$

Ans $Pr(E) = \dfrac{n(E)}{n(S)} = \dfrac{3!}{27}$

$= \dfrac{6}{27} = \dfrac{2}{9}$