11. p. 147 §6.4 12 boys, 10 girls. Pick 7 students.

What's the prob. of picking at least 2 girls?

Assume any set of 7 students is just as likely to be picked as any other seven.

Sample space set of all possible outcomes to the experiment. We want this to be equally likely outcomes.

S = { all choices of 7 people from the 22 }.

E = { all choices of 7 people from the 22 with 2 or more girls }.

↑ subset of S

$$Pr(E) = \frac{n(E)}{n(S)}.$$

Let's calculate!

$$n(S) = \binom{22}{7} = \frac{22\cdot 21\cdot 20\cdot 19\cdot 18\cdot 17\cdot 16}{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2} = 170544$$

$$n(E) = \binom{10}{2}\binom{12}{5} + \binom{10}{3}\binom{12}{4} + \binom{10}{4}\binom{12}{3} + \binom{10}{5}\binom{12}{2} + \binom{10}{6}\binom{12}{1} + \binom{10}{7}\binom{12}{0}$$

35640 — # of ways to choose 7 people, exactly 2 girls & 5 boys

59400 — # of sets with exactly 3 girls

46200 — exactly 4 girls

16632 — exactly 5 girls

2520 — exactly 6 girls

120 — exactly 7

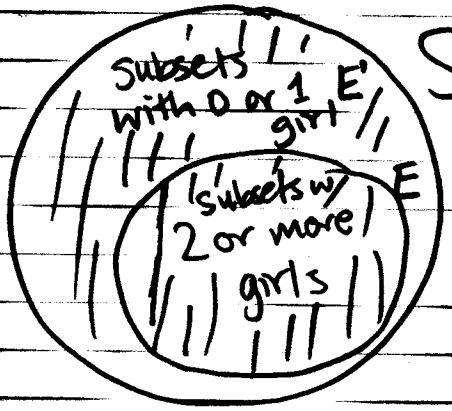
$$n(E) = 160{,}512$$

$$Pr(E) = \frac{160512}{170544} = .9412$$

Let's think of Pr(E) another way!

$$Pr(E) = 1 - Pr(E')$$

E' = { sets of 7 people with 0 or 1 girl }



We'll calculate $Pr(E')$.

$n(S) = 170544$ same.

$$n(E') = \binom{10}{0}\binom{12}{7} + \binom{10}{1}\binom{12}{6}$$

$\binom{10}{0}$: 1 girl choices; $\binom{12}{7}$: boy choices — # of ways to pick a set of 7 with no girls.

$\binom{10}{1}\binom{12}{6}$: ways to pick 7 with 1 girl & 6 boys

$= 792 + 9240$

$= 10,212$

$$Pr(E') = \frac{n(E')}{n(S)} = \frac{10,212}{170544}$$

$= .0588$

$Pr(E) + Pr(E') = 1$

$Pr(E) = 1 - Pr(E')$

$= 1 - .0588$

$= .9412$

Excel

p.147

14. What's the prob the first 3 are boys?

(still picking 7 from 22 with 12 boys & 10 girls).

Order matter!

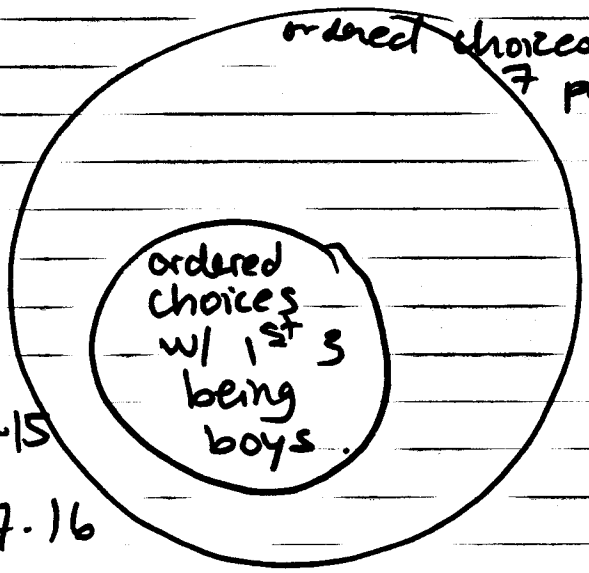
E = { picks of 7 people in a row where first 3 are boys }

Not really a subset of my original sample space S.

Change the sample space.

Let S = { ordered choices of 7 people }

Notice that E ⊂ S



$$Pr(E) = \frac{n(E)}{n(S)}$$

$n(S) = \cancel{22 \cdot 21 \cdot 20 \cdot 19 \cdot 18 \cdot 16 \cdot 15}$

$22 \cdot 21 \cdot 20 \cdot 19 \cdot 18 \cdot 17 \cdot 16$

$n(E) = 12 \cdot 11 \cdot 10 \cdot 19 \cdot 18 \cdot 17 \cdot 16$

$$Pr(E) = \frac{12 \cdot 11 \cdot 10}{22 \cdot 21 \cdot 20} = .1429$$

$S = \{$ ways to Pick 1st 3 kids, then last 4 kids $\}$

$E = \{$ ways to pick 1st 3 kids boys, then last 4 kids $\}$

$Pr(E) = \frac{n(E)}{n(S)}$ $\quad n(S) = \binom{22}{3} \cdot \binom{19}{4}$

($\binom{22}{3}$: picked 3 kids; $\binom{19}{4}$: picked 4 kids)

$n(E) = \binom{12}{3}\binom{19}{4}$

($\binom{12}{3}$: pick 3 boys; $\binom{19}{4}$: pick 4 kids of those left)

$$\frac{n(E)}{n(S)} = \frac{\binom{12}{3}\binom{19}{4}}{\binom{22}{3}\binom{19}{4}} = .1429$$

Even easier!

$S = \{$ ways to pick 3 kids (we'll say these are the "first" kids) $\}$

$E = \{$ ways to pick 3 boys (first kids) $\}$

$n(E) = \binom{12}{3}$

$n(S) = \binom{22}{3}$

Ans $Pr(E) = \frac{\binom{12}{3}}{\binom{22}{3}} = .1429.$

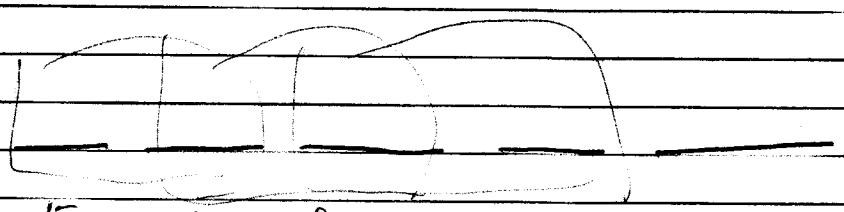
Problem Done in Office Hrs

§6.4

25 5 people, Mom, Dad, 3 kids.
take picture of people in a row

Q How many ways can Mom & Dad be adjacent?

$S = \{$ choice of how to order 5 people in a row $\}$

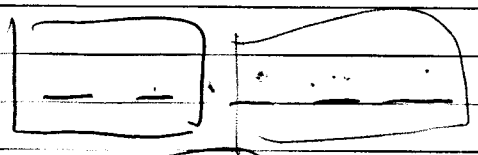


5 spots for people

$E = \{$ ways to order 5 people w/ parents together $\}$

4 possible spots for Mom & dad to be in picture:

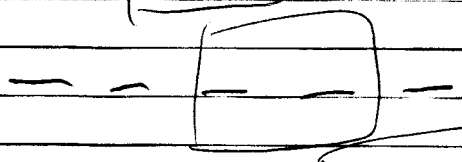
M D . . .
D M . . .



× 2

× 2

In 3 other spots
$3! = 3 \cdot 2 \cdot 1$
ways to put kids in remaining spots



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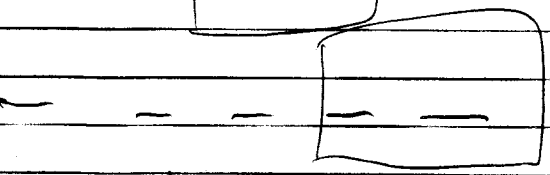

× 2

$n(E) = 4 \cdot 2 \cdot 3!$ = # of ways to compose the picture.

- 4 spots to put in parents
- 2 ways to order parents
- $3!$ ways to order kids

multiplying by 2 to indicate M & D vs D-M.

$n(S) = 5!$

Ans $\frac{4 \cdot 2 \cdot 3!}{5!} = \frac{8}{20} = \frac{2}{5}$