25(c) p.82

Put 9 people in order

Pitcher last
Catcher $8^{th}$
Shortstop $1^{st}$

Break task into operations

Op 1 : Pick $1^{st}$ player $m_1 = 1$ ← shortstop
Op 2 $2^{nd}$ player $m_2 = 6$ ← anyone but catcher, pitcher, shortstop
Op 3 $3^{rd}$ $m_3 = 5$
⋮ $m_4 = 4$
$m_5 = 3$
Op 9 : $9^{th}$ player $m_6 = 2$
$m_7 = 1$
Ans: $1 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 \cdot 1$ $m_8 = 1$ ← catcher
$m_9 = 1$ ← pitcher

Another way:

$m_1 = 1$ Op 1 Pick $1^{st}$, $8^{th}$, last batter as given
$m_2 = 6$ Op 2 Pick $2^{nd}$
$m_3 = 5$ Op 3 Pick $3^{rd}$
$m_4 = 4$ ⋮
$m_5 = 3$ Op 7 Pick $7^{th}$ batter
$m_6 = 2$
$m_7 = 1$ ← Same answer.
$1 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

37 (modified)

If everyone in winning team shakes (right) hands with everyone in losing team.
How many handshakes?

Op 1 Win Player 1 shakes all loser hands $m_1 = 10$
Op 2 Winning Player 2 - - - - - - - . $m_2 = 10$
⋮
Op 10 Winning Player 10 - - - - $m_{10} = 10$

Ans. $10 + 10 + 10 + \cdots + 10 = 100$ ?

Wait! Is this mult. principle? NO

Not MP because $m_2$ isn't the # of handshakes for each handshake of player 1.

Can we use MP?

<u>Op 1</u> Pick 1 player from winning team $m_1 = 10$

<u>Op 2</u> Pick 1 player from loser team. $m_2 = 10$
(Make them shake hands).

Notice: all the ways we could do this are all the handshakes.

Ans. 100 = 10·10

<u>Original problem:</u> Each winner shakes with <u>all</u> the other players

<u>Op 1</u> Pick 1 player from winning team $m_1 = 10$

<u>Op 2</u> Pick 1 of remaining players $m_2 = 19$

Ans. 10·19 = 190

p.90

29. Own 10 sweaters

How many ways to pick 6 to leave at home?

Order doesn't matter: $C(10,6)$ "10 choose 6"

$$= \frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1} \leftarrow \text{6 terms}$$

10 = how many you are choosing from (10 sweaters).

6 = how many you are choosing (6 sweaters).

Why this formula?

Pick 6 sweaters from 10 and put them in order = <u>Op 1</u> Pick 6 sweaters from 10

<u>Op 2</u> Put the 6 sweaters in order

$$P(10,6) = C(10,6)\cdot \underset{\substack{\uparrow\\ \text{6 choices ... for 1st}}}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}$$

$$= \underset{\substack{\uparrow\\ \text{10 choices for 1st}}}{10}\cdot 9\cdot 8\cdot 7\cdot 6\cdot \underset{\substack{\uparrow\\ \text{5 choices for 6th sweater}}}{5}$$

$$\frac{P(10,6)}{6!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1} = C(10,6)$$

But isn't picking 6 sweaters to leave at home the same as picking 4 sweaters to take with you?

$$C(10,6) \stackrel{?}{=} C(10,4)$$

$$\frac{10\cdot 9\cdot 8\cdot 7\cdot \cancel{6}\cdot \cancel{5}}{\cancel{6}\cdot \cancel{5}\cdot 4\cdot 3\cdot 2\cdot 1} = \frac{10\cdot 9\cdot 8\cdot 7}{4\cdot 3\cdot 2\cdot 1}$$

yes!

$$C(n,k) = \frac{\overbrace{n(n-1)\cdots(n-k+1)}^{n \text{ terms}}}{k\cdot(k-1)\cdots 1}$$

$$\| $$

$$C(n,n-k) = \frac{n(n-1)\cdots(n-(n-k)+1)}{(n-k)(n-k-1)\cdots 1} \leftarrow n-k \text{ terms}$$

wow!

\# of ways to choose k objects from n
\# of ways to leave out n−k objects

(You can choose k, or choose n−k, and you have same result ∴ i.e. the same # of ways to do this).

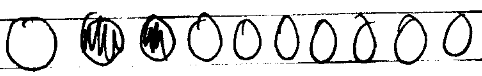
Ex Pick a committee of 8 students from a group of 9. How many ways?

$$C(9,8) = \frac{9\cdot\cancel{8}\cdot\cancel{7}\cdot\cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}}{\cancel{8}\cdot\cancel{7}\cdot\cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1} = 9$$

$$C(9,1) = \frac{9}{1} = 9$$

Flip a coin 10 times.
How many ways can I get exactly 2 H ?

\# of ways to get 2 H = \# of ways to pick 2 of the ten flips (and assign them H, while others are T).

 $\longleftrightarrow$ Pick $2^{nd}$ & $3^{rd}$ flips

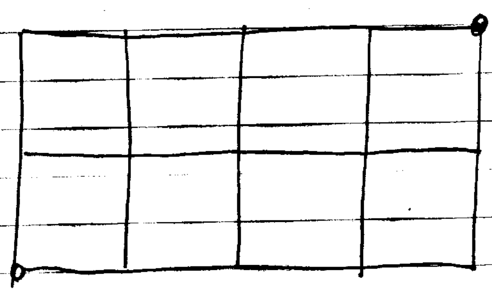
we can count this!

10 flips: choose 2 of them.
We only care about the set of the 2 flips (2nd & 3rd flip is same as 3rd & 2nd flip).

$$C(10,2) = \frac{10 \cdot 9}{2 \cdot 1}$$

COMBINATORICS: STUDY OF COUNTING

A



B

To go from point A to B along the grid, you can only go down or left at each move.

How many ways can you get from A to B?

There are always 6 moves. Always 2 down moves.

$$C(6,2) = \frac{6 \cdot 5}{2 \cdot 1} = 15$$

6 moves ; pick 2 that are down moves

$$C(6,4) = \frac{6 \cdot 5 \cdot \cancel{4} \cdot \cancel{3}}{\cancel{4} \cdot \cancel{3} \cdot 2 \cdot 1} = 15$$

pick which are left moves

Combining Mult ~~&~~ Principle & ~~the~~ Combinations / Permuations.

Organizing a science competition.

I have to pick 5 students from 15 to compete and 3 events from 10 to compete ~~about~~ on. (All 5 students work on all 3 events).
How many ways can I ~~do~~ do this?

Op 1 Pick students
Op 2 Pick events.

$\Big\}$ Use mult principle.

$\Rightarrow$ The answer is $m_1 \cdot m_2$ where

$m_1$ = # of ways to pick 5 students from 15.
$m_2$ = # of ways to pick 3 events from 10.

$$m_1 = C(15,5) = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$m_2 = C(10,3) = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1}$$

Ans $\frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \cdot \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1}$

$(x+y)^n = \text{Ack!}$

n integer $\quad (x+y)^2 = x^2 + 2xy + y^2$

$(x+y)^3 = (x+y)^2(x+y) = (x^2+2xy+y^2)(x+y)$

$\cancel{(x+y)(x+y)(x+y)}$ $\quad = x^3 + 2x^2y + x^2y$

$= xxx + xyy + yxy + yyx$

$+ xxy + xyx + yxx$ $\quad + xy^2 + 2xy^2 + y^3$

$\underline{C(n,0) = 1}$

$= x^3 + 3x^2y + 3xy^2 + y^3$

coeff. is $C(3,0)$ $\quad C(3,1)$ $\quad C(3,2)$ $\quad C(3,3)$

$(x+y)^4 = ?$ Ack

$(x+y)(x+y)(x+y)(x+y)$ $\quad = x^4 + 4x^3y$

$+ 4y^3x$

$x^2y^2$

Each $x + y$ ~~will be used~~ 'term will contribute either $x$ or $y$ to ~~the~~ each monomial

$(x+y)(x+y)(x+y)(x+y) \cdots (x+y)$

Pick $k$ of these that contribute $x$

$(x+y)^n = \cancel{\binom{n}{k} x^k y^{n-k}}$

$$\sum_{k=0}^{n} C(n,k)\, x^k y^{n-k}$$

SUM

# of ways to pick which terms will count $X$

$C(6,4) = C(6,2)$ $\qquad$ $C(n,k) = C(n, n-k)$

$$(x+y)^6 = C(6,0)\,x^0 y^{6-0} + C(6,1)\,x^1 y^{6-1}$$
$$+ C(6,2)\,x^2 y^{6-2} + C(6,3)\,x^3 y^{6-3}$$
$$+ C(6,4)\,x^4 y^{6-4} + C(6,5)\,x^5 y^{6-5}$$
$$+ C(6,6)\,x^6 y^{6-6}$$

$$= \boxed{y^6 + 6xy^5 + 15x^2y^4 + 20x^3y^3 + 15x^4y^2 + 6x^5y + x^6}$$

$C(6,1) = \frac{6}{1} = C(6,5)$

$C(6,2) = \frac{6 \cdot 5}{2 \cdot 1}$

$C(6,3) = \frac{\cancel{6} \cdot 5 \cdot 4}{\cancel{3} \cdot \cancel{2} \cdot 1} = 20$

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

$C(n,k) = C(n-1,k-1) + C(n-1,k)$

COEFFICIENTS

0th, 1st, 2nd, 3rd

0th row → 1

1 1

1 2 1

1 3 3 1

4th row → 1 4 6 4 1

5th row → 1 5 10 10 5 1

6th row → 1 6 15 20 15 6 1

1 7 21 35 35 21 7