Exploratory Hall 4214 → pick up exams.

WARM UP

Flip a coin, then roll a die & record the flip & the top face value.

Sample space = ALL POSSIBLE OUTCOMES = { H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6 } = S

Equally likely? YES!

Calculate prob. that you got Heads, or Tails with odd #.

E = event = subset of outcomes of interest
= { H1, H2, H3, H4, H5, H6, T1, T3, T5 }

$$Pr(E) = \frac{n(E)}{n(S)} = \frac{9}{12} = \frac{3}{4}$$

↗ equally likely outcomes.

Note: if F = event of heads = {H1, H2, H3, H4, H5, H6}

G = event of tails with odd # = {T1, T3, T5}

Then $Pr(F) = \frac{1}{2}$, $Pr(G) = \frac{1}{4}$

$$Pr(E) = Pr(F) + P(G)$$
$$\frac{3}{4} = \frac{1}{2} + \frac{1}{4}$$

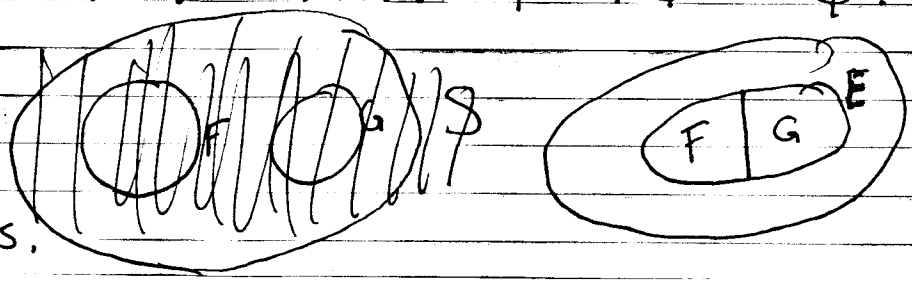
When the events don't intersect, we can add probabilities!

we also have $E = F \cup G$

But.... $F \cap G = \phi$.

Venn diagram

F and G are mutually exclusive events.



#3 p. 139

Roll 2 dice: red & green

36 equally likely outcomes

Sum to 8: $E = \{(2,6), (3,5), (6,2), (4,4), (5,3)\}$ 5 elements

Ans $\frac{5}{36} = Pr((2,6)) + Pr((3,5)) + Pr((6,2)) + Pr((4,4)) + Pr((5,3))$

$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36}$

#10 p. 139 $E = \{s_1, s_2\}$ $Pr(E) = .3$

Let $G = \{s_2, s_3\}$ $Pr(G) = .3$

$Pr(G \cup E) \neq Pr(G) + Pr(E) = .6$

$= Pr(\{s_1, s_2, s_3\}) = .05 + .25 + .05$

$= .35$

$= Pr(G) + Pr(E) - Pr(E \cap G)$

$= .3 + .3 - .25$

$= .35$ ✓✓

Princ. of Inclusion / Exclusion for Probability:

Let E and F be any events.

$$Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E \cap F).$$

If E and F are mutually exclusive, $E \cap F = \phi$

$$\Rightarrow \quad Pr(E \cup F) = Pr(E) + Pr(F).$$

Ex Roll a 20-sided die & record the face value.

What's the prob. it's an odd # less than, or equal to, 7 ?

Let E = event the # is odd

F = event it's ≤ 7.

PIE

$$Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E \cap F)$$

Not a good example to use PIE b/c we don't know either $Pr(E \cup F)$ or $Pr(E \cap F)$.

$Pr(E) = \frac{1}{2}$

$Pr(F) = \frac{7}{20}$

$Pr(E \cap F) = \frac{n(E \cap F)}{20}$ ← # of rolls ≤ 7 and odd

20 ← 20 equally likely outcomes

$= \frac{4}{20} = \frac{1}{5}$

Then $E \cup F = \{ \leq 7 \text{ or odd} \}$

$$Pr(E \cup F) = \frac{1}{2} + \frac{7}{20} - \frac{1}{5} = \frac{10}{20} + \frac{7}{20} - \frac{4}{20} = \frac{13}{20}$$

Def The odds of an event:

Ratio: $\frac{\text{probability of event}}{\text{probability of the event not occurring.}}$

Ex You flip 2 coins. What are the odds of getting 2 heads? $S = \{HH, HT, TH, TT\}$

$Pr(\{HH\}) = \frac{1}{4}$

Odds $\frac{1/4}{3/4} = \frac{1}{3}$

$$\text{Odds of } E = \frac{P(E)}{1 - P(E)}$$ ← given P(E), you can find the odds

Then if you're given the odds, you can also find the probability!

Ex Odds $\frac{5}{8}$ ⇝ Prob $\frac{5}{13}$ ← E

E, E' (odds $5:8$)

$E \cup E' = S$ (prob $5:13$)

#4 p. 136 $S = \{50 \text{ states}\}$ $E = \{6 \text{ NE states}\}$ $Pr(E) = \frac{n(E)}{n(S)} = \frac{6}{50} = \frac{3}{25}$

Pick 2 students from a group of 10 students containing Kevin.
What's the prob. I pick Kevin?

$S = \{\text{choices of pairs of students}\}$ $|S| = C(10,2) = \frac{10 \cdot 9}{2 \cdot 1} = 45$

$E = \{\text{pairs of students with Kevin}\}$ $|E| = 9$

↑ pick Kevin and one other student

Ans $\frac{9}{45} = \frac{1}{5}$

#8 p. 147 An urn contain 6 white & 5 red balls.
A sample of 4 balls is chosen.
What's the prob of getting 2 white & 2 red balls?

Want equally likely outcomes: $S = \{\text{samples of 4 balls}\}$ $|S| = \binom{11}{4} = C(11,4)$

↑ 11 balls, choose 4

$E = \{\text{samples of 4 balls with 2 white \& 2 red}\}$

$|E| = \binom{5}{2}\binom{6}{2}$

↑ ways to pick 2 red ↑ ways to pick 2 white

Ans $\frac{\binom{5}{2}\binom{6}{2}}{\binom{11}{4}}$