#11 p. 167

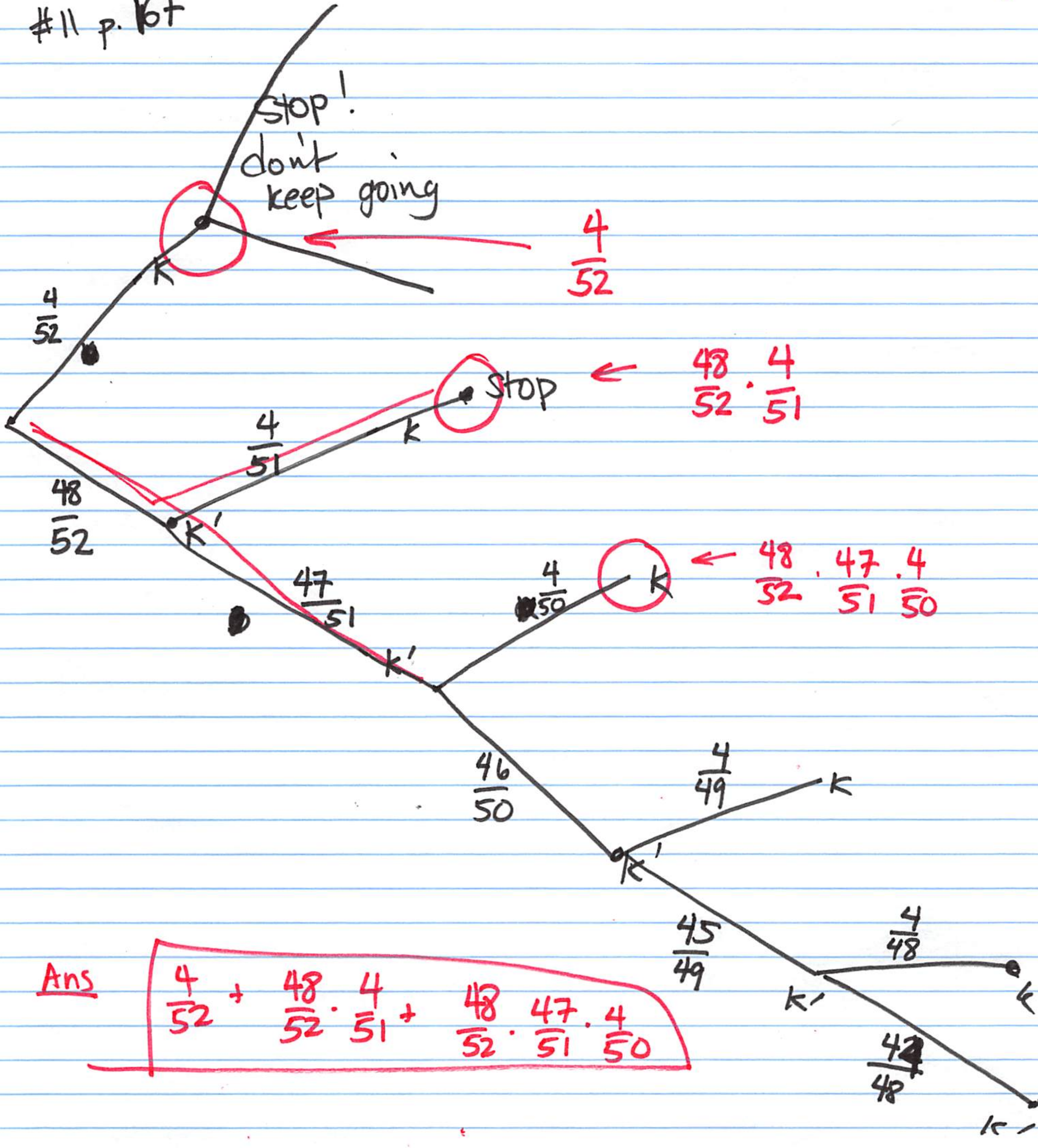
$\frac{4}{52}$

$\frac{48}{52} \cdot \frac{4}{51}$

$\frac{48}{52} \cdot \frac{47}{51} \cdot \frac{4}{50}$

Ans $\boxed{\frac{4}{52} + \frac{48}{52} \cdot \frac{4}{51} + \frac{48}{52} \cdot \frac{47}{51} \cdot \frac{4}{50}}$

#11 p. 167



Ans $\boxed{\frac{4}{52} + \frac{48}{52}\cdot\frac{4}{51} + \frac{48}{52}\cdot\frac{47}{51}\cdot\frac{4}{50}}$

Example

Testing for disease:

B_1 = person has TB

B_2 = person does not have TB.

A = positive skin test result.

Suppose .02% of a population has TB. (2 in 10000)

$Pr(B_1)$

[The skin test has a false negative rate of 2% $Pr(A'|B_1) = .02$

[and a false positive rate of 1%.

$Pr(A|B_2)$

What is the chance that someone who tests positive actually has TB? $Pr(B_1|A)$

Guess Ans
a) 50-90%
b) 90 to 100%
c) 10-50%
d) 5-10%
e) <5%

Calculate

$Pr(B_1) = .0002$

$Pr(B_2) = .9998$

$Pr(A'|B_1) = .02$

$Pr(A|B_1) = .98$

$Pr(A|B_2) = .01$

WANT $Pr(B_1|A)$

$$Pr(B_1|A) = \frac{Pr(A|B_1)Pr(B_1)}{Pr(A|B_1)Pr(B_1) + Pr(A|B_2)Pr(B_2)}$$

$$= \frac{.98 \cdot .0002}{.98 \cdot .0002 + .01 \cdot .9998}$$

$$\approx .02$$

approximate

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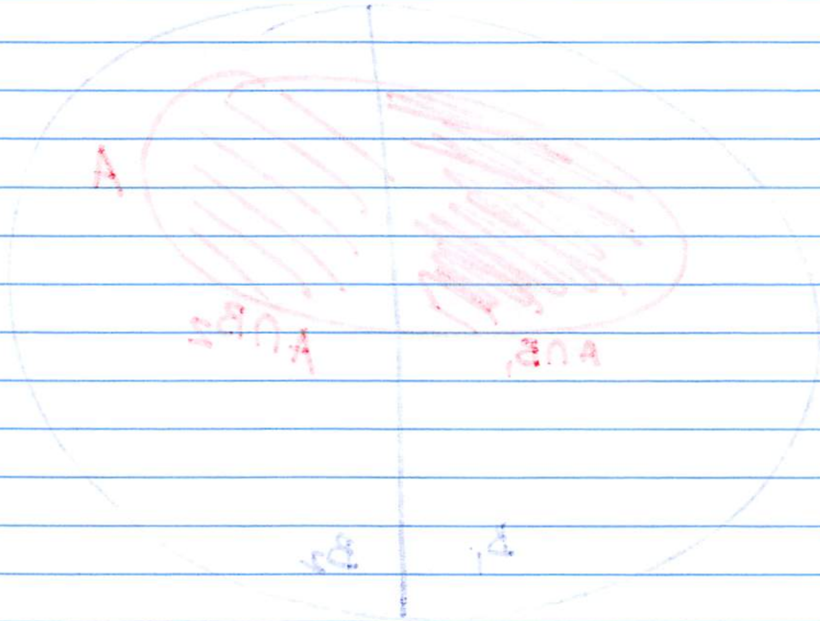
Back to original question



$Pr(A \cap B_1) = Pr(B_1|A)Pr(A)$
$= Pr(A|B_1)Pr(B_1)$

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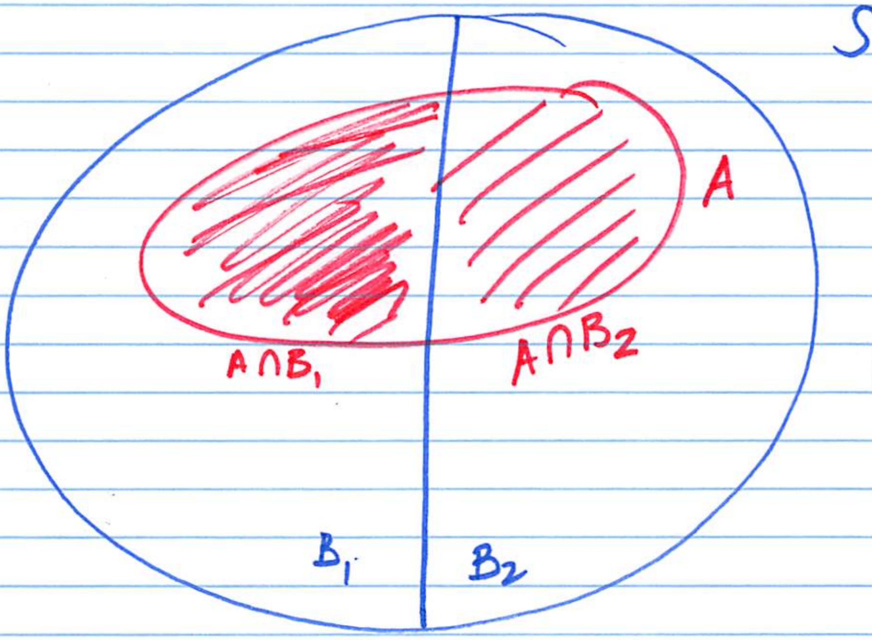
Bayes' Thm (2 sets) If A, B_1, B_2 are events with $B_1 \cup B_2 = S$ and $B_1 \cap B_2 = \emptyset$ then

$$Pr(B_1 | A) = \frac{Pr(A|B_1)Pr(B_1)}{Pr(A|B_1)Pr(B_1) + Pr(A|B_2)Pr(B_2)}$$

Back to original question



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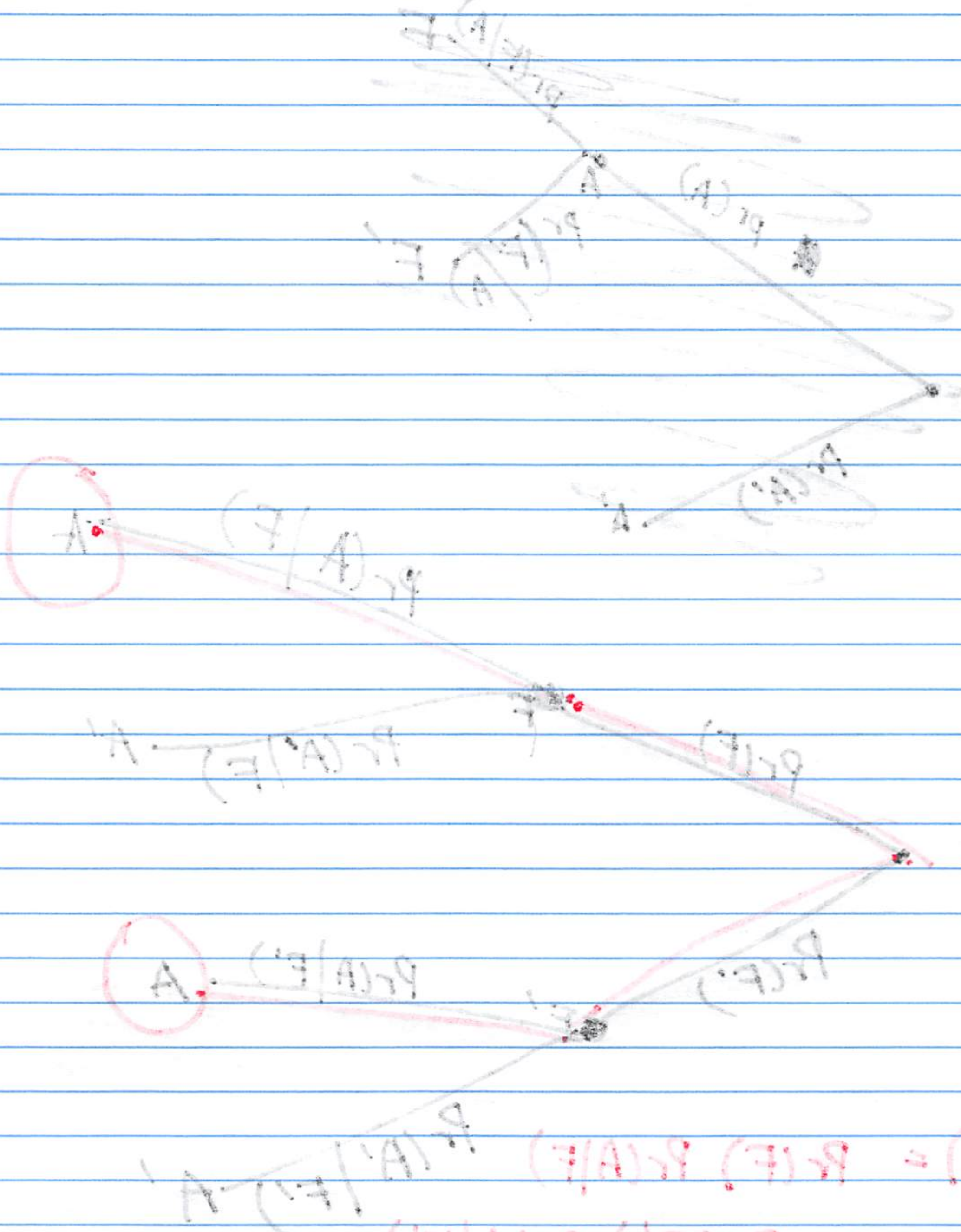
4

$Pr(A) = Pr(A|F)\,Pr(F) + Pr(A|F')\,Pr(F')$

Tree picture



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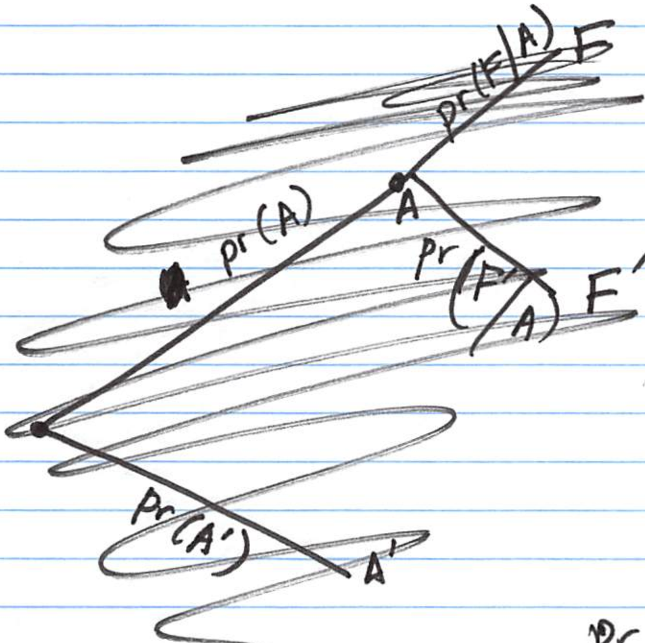

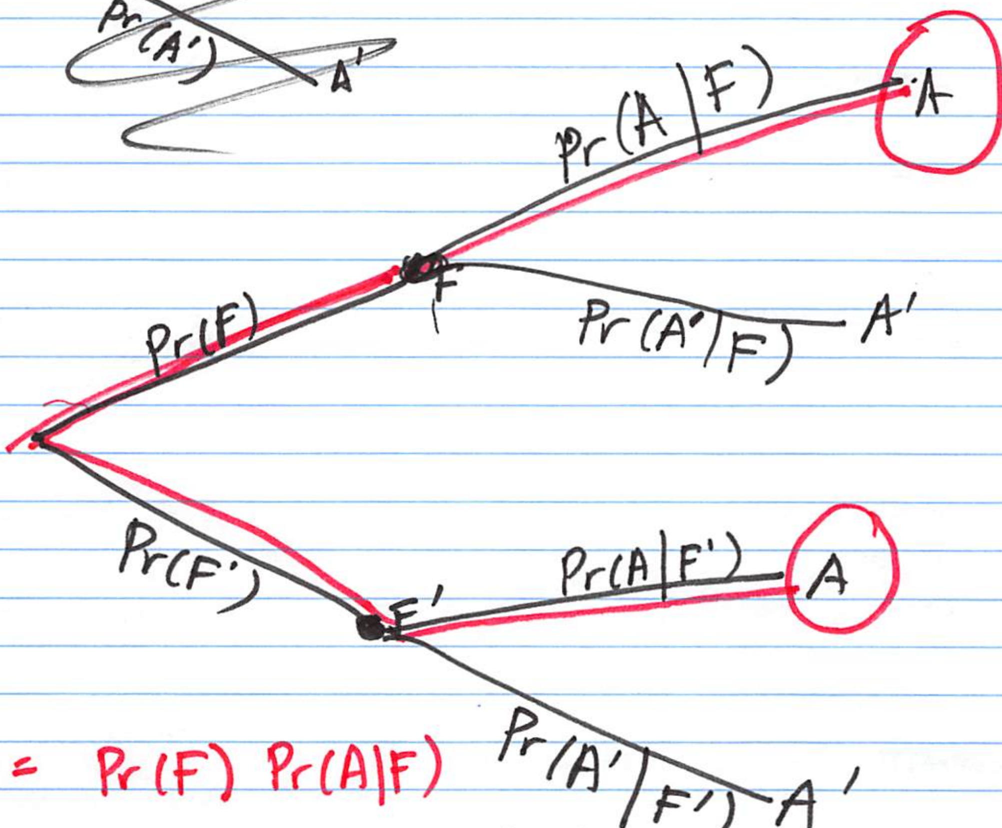

$$Pr(A) = Pr(F)\,Pr(A|F) + Pr(F')\,Pr(A|F')$$

Ex Let's suppose over the course of a year, a car sales dealer sells 2000 cars. Of these, 550 were sold on Saturdays. Also, women bought 68% of the cars on all days. What information would you need to find the probability that a car sold on Saturday was sold to a woman?

S = people buying cars (with dates & genders).
F = people buying cars on Saturdays.
A = females buying cars

$$\underbrace{Pr(F)}_{\substack{\text{Known} \\ \frac{550}{2000}}} = \underbrace{Pr(F \mid A) \cdot Pr(A)}_{\text{prob. of being female and buying car on Saturday}} + \underbrace{Pr(F \mid A')Pr(A')}_{\text{prob. of being male \& buying car of Saturday}}$$

not what we wanted.

We wanted $Pr(A \mid F)$

~~this is prob.~~

$$Pr(A) = \underbrace{Pr(A \mid F)}_{\text{This is what we want.}} Pr(F) + Pr(A \mid F')Pr(F')$$

$Pr(A)$: this is known .68

$Pr(F)$: Known $\frac{550}{2000}$

$Pr(A \mid F')$: If we knew this we could solve for $Pr(A \mid F)$

$Pr(F')$: Known $1 - \frac{550}{2000}$

Ans If we knew the prob. of a car sold on any day except Saturday was sold to a woman, we could get the prob. it was a car of Sat. was sold to a woman.

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Some algebra...

$$\Pr(E|F) = \frac{\Pr(E\cap F)}{\Pr(F)} \Rightarrow \boxed{\Pr(E\cap F) = \Pr(E|F)\Pr(F)}$$

MEMORIZE

$$\|$$

$$\Pr(F|E) = \frac{\Pr(F\cap E)}{\Pr(E)} \Rightarrow \Pr(F\cap E) = \Pr(F|E)\cdot \Pr(E)$$

$$F\cap E = E\cap F$$

(1) $\boxed{\Pr(E|F)\Pr(F) = \Pr(F|E)\Pr(E)}$

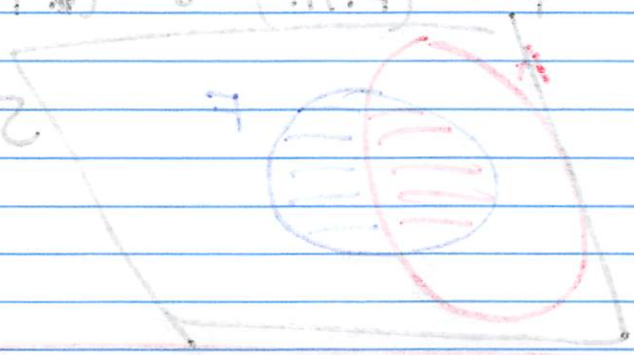
Let's suppose there's another event A.

$$\Pr(F) = \Pr(A)\Pr(F|A) + \Pr(A')\Pr(F|A')$$

Why?

$$\Pr(F) = \Pr(F\cap A) + \Pr(F\cap A')$$

(Note: $F = (F\cap A) \cup (F\cap A')$)



(2) $\boxed{\Pr(F) = \underbrace{\Pr(F|A)\Pr(A)}_{=\Pr(F\cap A)} + \underbrace{\Pr(F|A')\Pr(A')}_{\Pr(F\cap A')}}$

(1) and (2) follow from previous material.

Some algebra...

MEMORIZE

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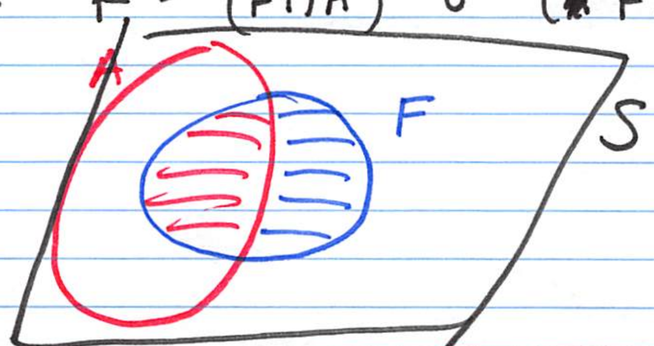

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① and ② follow from previous material.

Today: Bayes' Thm (Ch 6.7) 4/13

Recall If E, F are two events,

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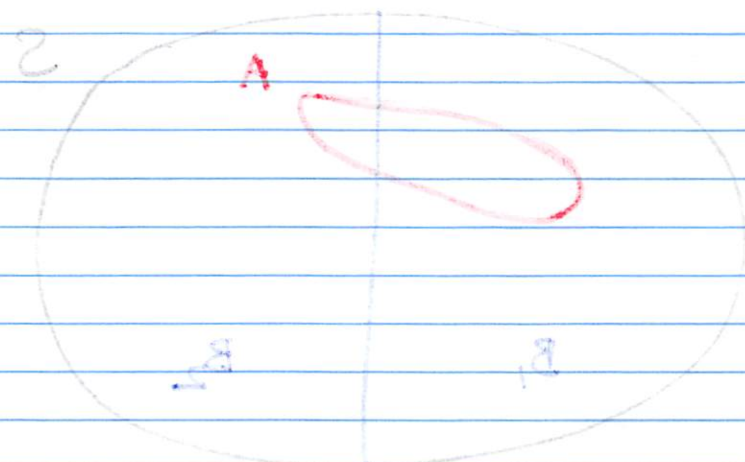
$$Pr(F \mid E) = \frac{Pr(E \cap F)}{Pr(E)} \Rightarrow Pr(E \cap F) = Pr(F \mid E) \cdot Pr(E)$$

Bayes' is sort of a generalization...

Let's suppose A is an event in S

and B_1, B_2 are mutually exclusive events

such that $B_1 \cup B_2 = S$.



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What if we don't know these probabilities on the right?

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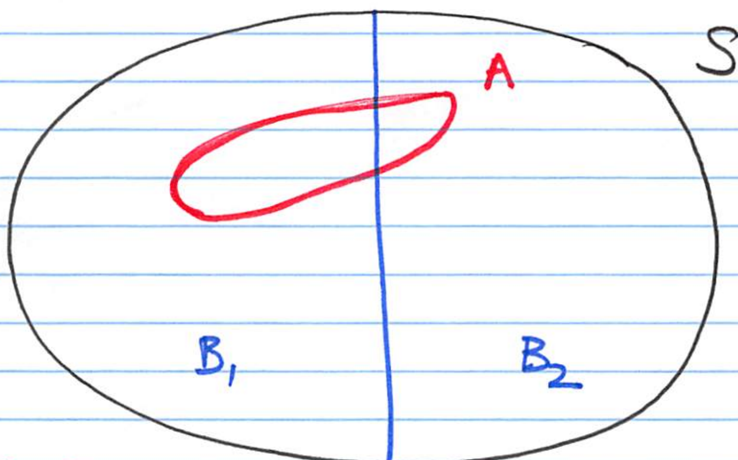

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