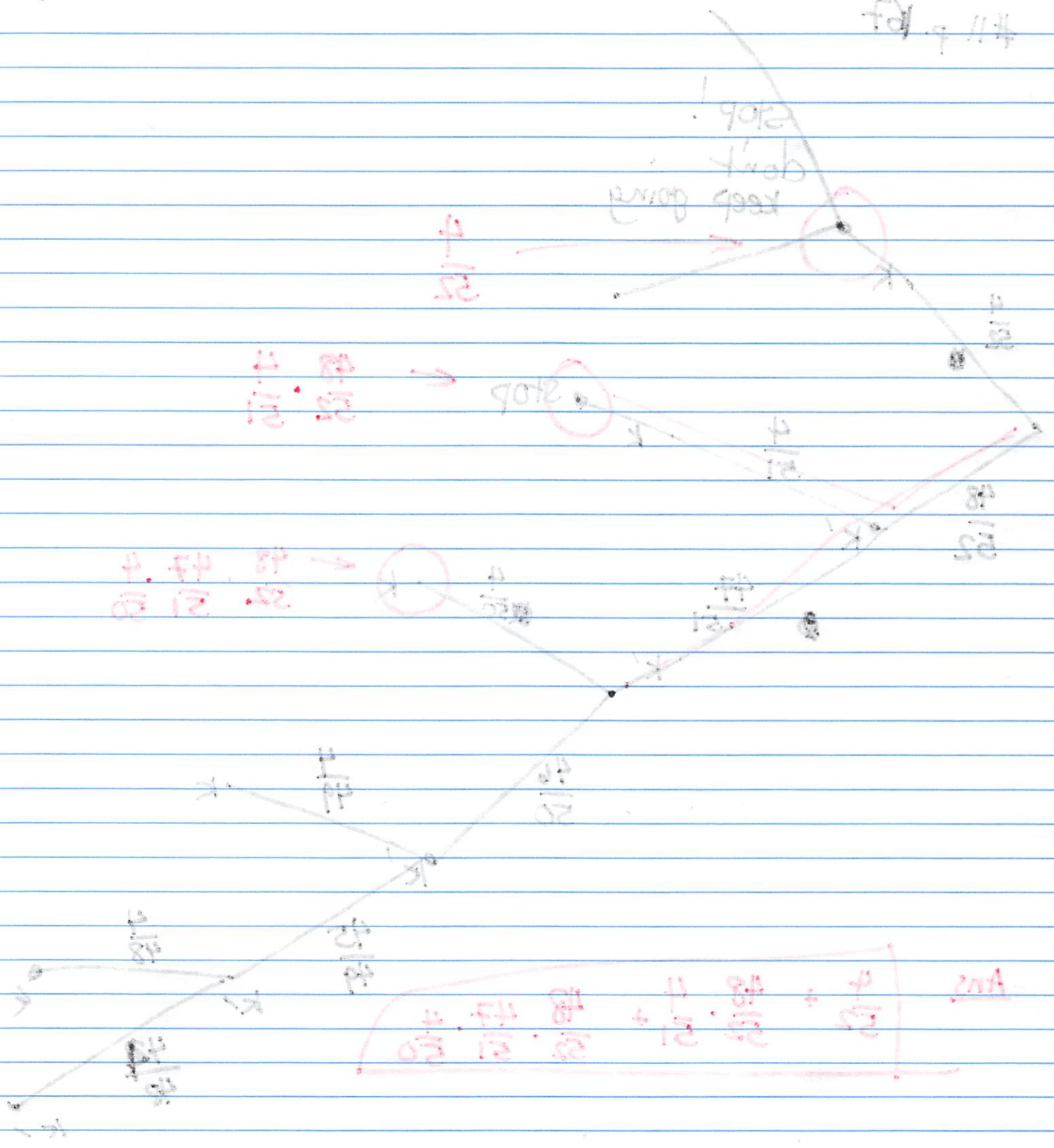
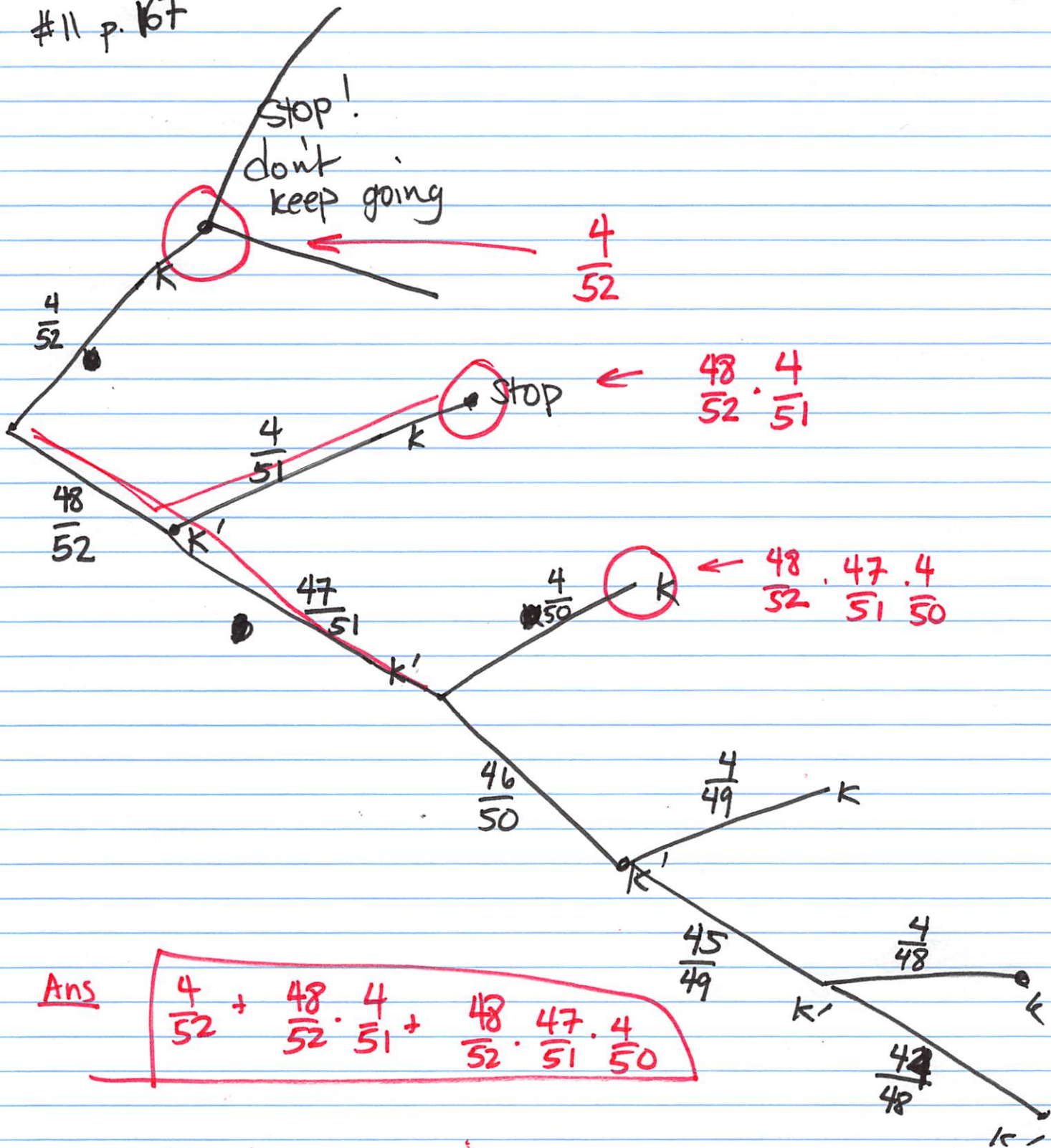


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#11 p. 167



Ans

$$\frac{4}{52} + \frac{48}{52} \cdot \frac{4}{51} + \frac{48}{52} \cdot \frac{47}{51} \cdot \frac{4}{50}$$

Example

Testing for disease:

B_1 = person has TB

B_2 = person does not have TB

A = positive skin test result

Suppose 0.5% of a population has TB. (5 in 1000)

The skin test has a false negative rate of 2% $P(A|B_1) = 0.02$

and a false positive rate of 1% $P(A|B_2)$

What is the chance that someone who tests positive actually has TB?

Calculate

$P(B_1) = 0.005$

$P(B_2) = 0.995$

$P(A|B_1) = 0.02$

$P(A|B_2) = 0.01$

$P(A) = 0.01$

WANT $P(B_1|A)$

$P(B_1|A) = \frac{P(A|B_1)P(B_1)}{P(A)}$

$\frac{P(A|B_1)P(B_1)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2)}$

$$\frac{0.005 \cdot 0.02}{0.005 \cdot 0.02 + 0.995 \cdot 0.01}$$

approximate 0.2

- Guess
- a) 50-100%
 - b) 0 to 100%
 - c) 10-20%
 - d) 2-10%
 - e) < 2%

Example

Testing for disease:

 $B_1 =$ ~~person has TB~~ person has TB $B_2 =$ person does not have TB. $A =$ positive skin test result.

Suppose .02% of a population has TB. (2 in 10,000)

$\left[\begin{array}{l} \text{The skin test has a false negative rate} \\ \text{of } 2\% \quad \Pr(A' | B_1) = .02 \end{array} \right.$

$\left[\text{and a false positive rate of } 1\% \right.$

What is the chance that someone who tests positive
 actually has TB? $\Pr(B_1 | A)$

- Guess Ans
- a) 50-90%
 - b) 90 to 100%
 - c) 10-50%
 - d) 5-10%
 - e) <5%

Calculate

$$\Pr(B_1) = .0002$$

$$\Pr(B_2) = .9998$$

$$\Pr(A' | B_1) = .02$$

$$\Pr(A | B_1) = .98$$

$$\Pr(A | B_2) = .01$$

WANT $\Pr(B_1 | A)$

$$\Pr(B_1 | A) = \frac{\Pr(A | B_1) \Pr(B_1)}{\Pr(A | B_1) \Pr(B_1) + \Pr(A | B_2) \Pr(B_2)}$$

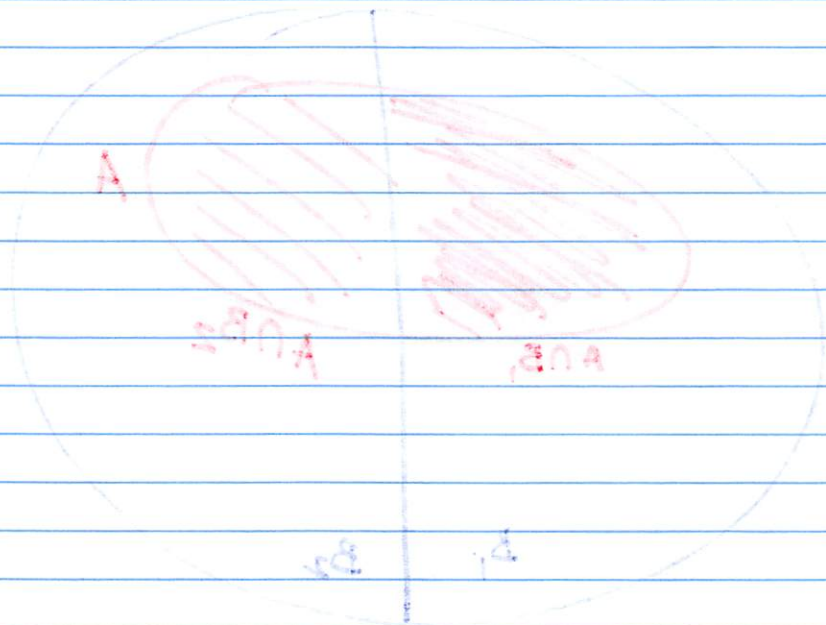
$$= \frac{.98 \cdot .0002}{.98 \cdot .0002 + .01 \cdot .9998}$$

$$\approx .02$$

↑
approximate.

Back to original question

2.



$$P(A \cap B) = P(B|A)P(A)$$

$$= P(A|B)P(B)$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A|B)P(B)}{P(A)}$$

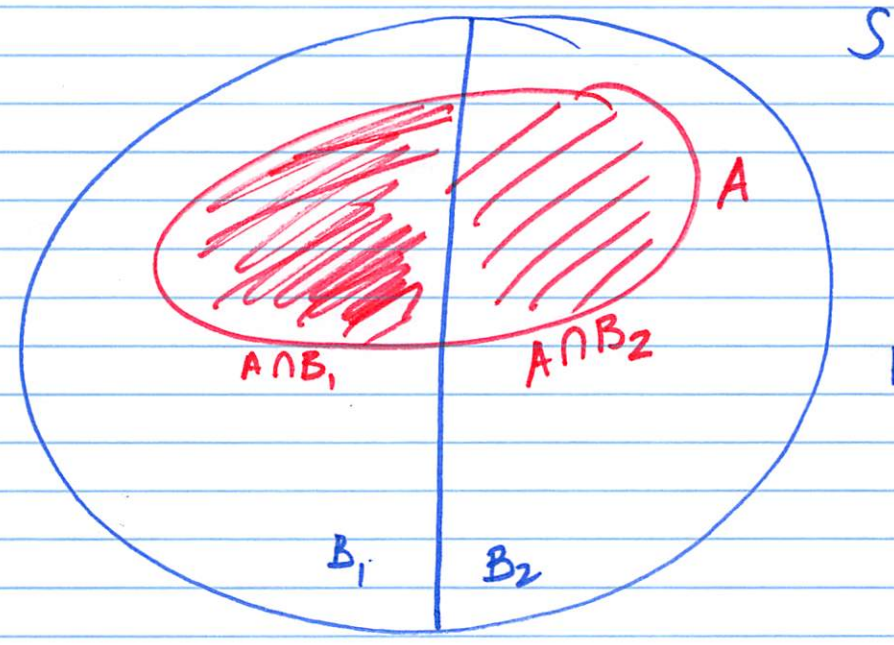
$$= \frac{P(A \cap B) + P(A \cap B^c)}{P(A \cup B^c)}$$

$$= \frac{P(A \cap B) + P(A|B^c)P(B^c)}{P(A \cup B^c)}$$

Bayes' Theorem (2 sets) If A, B, \bar{A} are events with $B, \bar{B} = \Omega$ and $B, \bar{B} = \emptyset$ then

$$P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|\bar{B})P(\bar{B})}$$

Back to original question



$$\begin{aligned} \Pr(A \cap B_1) &= \Pr(B_1 | A) \Pr(A) \\ &= \Pr(A | B_1) \Pr(B_1) \end{aligned}$$

$$\begin{aligned} \Pr(B_1 | A) &= \frac{\Pr(A \cap B_1)}{\Pr(A)} = \frac{\Pr(A | B_1) \Pr(B_1)}{\Pr(A)} \\ &= \frac{\Pr(A | B_1) \Pr(B_1)}{\underbrace{\Pr(A | B_1) \Pr(B_1)}_{\Pr(A \cap B_1)} + \underbrace{\Pr(A | B_2) \Pr(B_2)}_{\Pr(A \cap B_2)}} \end{aligned}$$

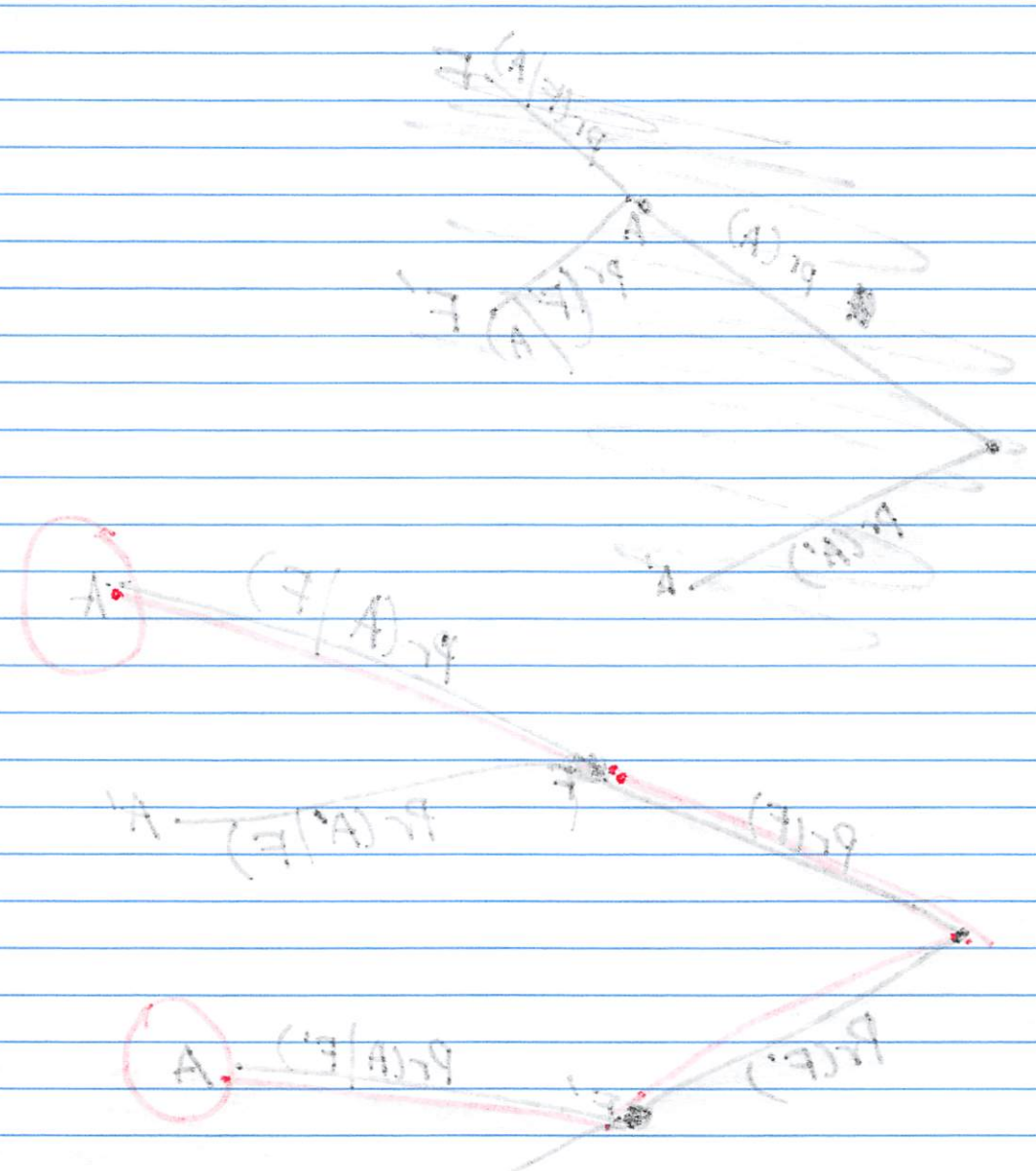
Bayes' Thm (2 sets) If A, B₁, B₂ are events with B₁ ∪ B₂ = S and B₁ ∩ B₂ = ∅ then

$$\Pr(B_1 | A) = \frac{\Pr(A | B_1) \Pr(B_1)}{\Pr(A | B_1) \Pr(B_1) + \Pr(A | B_2) \Pr(B_2)}$$

4

$$P(A) = P(A|F)P(F) + P(A|F')P(F')$$

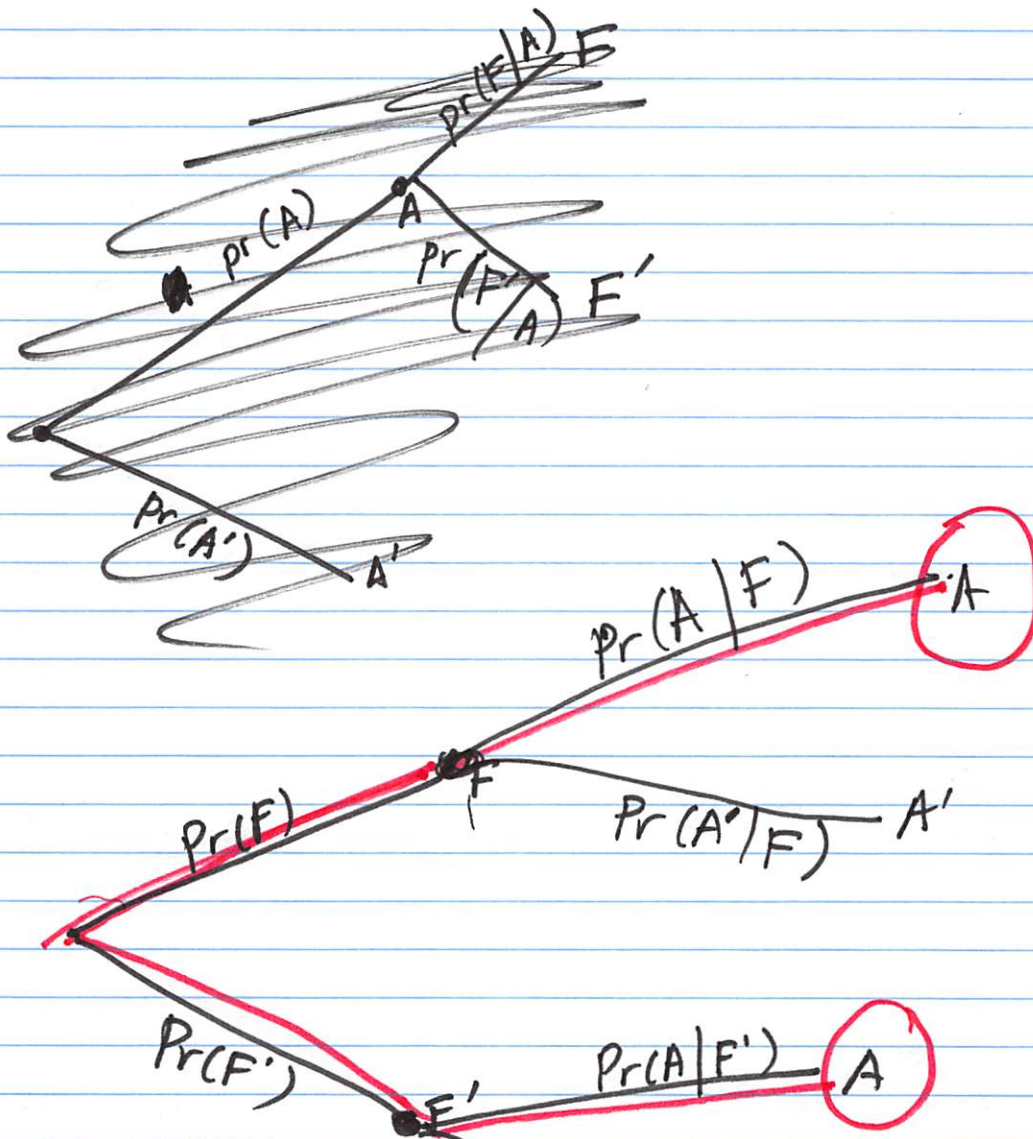
Tree Diagram



$$P(A) = P(A|F)P(F) + P(A|F')P(F')$$

$$Pr(A) = Pr(A|F) Pr(F) + Pr(A|F') Pr(F')$$

Tree picture



$$Pr(A) = Pr(F) Pr(A|F) + Pr(F') Pr(A|F')$$

Ex: let's suppose to give the context of

a dealer, a sales dealer sells 2000 cars. Of these, 220 were sold on Saturdays.

Also, women bought 88% of the cars on all days. What information would you need to find the probability that a car sold on Saturday was sold to a woman?

Let's define events:
 A = females buying cars
 F = people buying cars on Saturdays
 ? = people buying cars (with dates & genders)

$$P(A) = P(A|F) \cdot P(F) + P(A|\bar{F}) \cdot P(\bar{F})$$

$P(A) = 0.88$
 $P(F) = \frac{220}{2000}$
 $P(\bar{F}) = \frac{1780}{2000}$
 $P(A|F) = ?$
 $P(A|\bar{F}) = ?$

$$P(A) = P(A|F) \cdot P(F) + P(A|\bar{F}) \cdot P(\bar{F})$$

This is known (0.88)
 This is known (220/2000)
 This is what we want to find
 This is known (1780/2000)

Ans: If we know the prop. of a car sold on any day except Saturday, we could find the prob. that a car sold on Saturday was sold to a woman.

Ex Let's suppose ~~over~~ the course of a year, a ^{car} Sales dealer sells 2000 cars. Of these, 550 were sold on Saturdays.

Also, women bought 68% of the cars on all days. What information would you need to find the probability that a car sold on

Saturday was sold to a ~~man~~ woman?

S = people buying cars (with dates & genders).

F = ~~car~~ people buying cars on Saturdays.

A = females buying cars

$$\Pr(F) = \Pr(F|A) \cdot \Pr(A) + \Pr(F|A') \Pr(A')$$

Known
 $\frac{550}{2000}$

not what we wanted.
We wanted $\Pr(A|F)$

~~This is prob.~~

prob. of being male & buying car on Saturday

$$\Pr(A) = \Pr(A|F) \Pr(F) + \Pr(A|F') \Pr(F')$$

this is known
.68

This is what we want.

Known
 $\frac{550}{2000}$

If we knew this we could solve for $\Pr(A|F)$

Known
 $1 - \frac{550}{2000}$

Ans If we knew the prob. of a car sold on any day except Saturday ~~the~~ was sold to a woman, we could get the prob. ~~the~~ was a car of Sat. was sold to a woman.

Some algebra ...

MEMORISE

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$E \cup F = E \cup \bar{E} \cap F$$

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

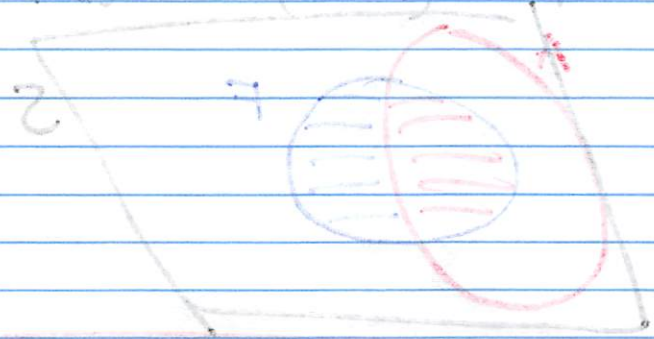
Let's suppose there's another event A.

$$P(E) = P(A)P(E|A) + P(\bar{A})P(E|\bar{A})$$

Why?

$$P(E) = P(E \cap A) + P(E \cap \bar{A})$$

(Note: $E = (E \cap A) \cup (E \cap \bar{A})$)



$$P(E) = P(E \cap A) + P(E \cap \bar{A})$$

$$P(E \cap A) = P(A)P(E|A)$$

(1) and (2) follow from previous material.

Some algebra...

$$\Pr(E|F) = \frac{\Pr(E \cap F)}{\Pr(F)}$$

⇒

MEMORIZE

$$\Pr(E \cap F) = \Pr(E|F) \Pr(F)$$

||

$$\Pr(F|E) = \frac{\Pr(F \cap E)}{\Pr(E)}$$

⇒

$$\Pr(F \cap E) = \Pr(F|E) \Pr(E)$$

$$F \cap E = E \cap F$$

$$\textcircled{1} \quad \Pr(E|F) \Pr(F) = \Pr(F|E) \Pr(E)$$

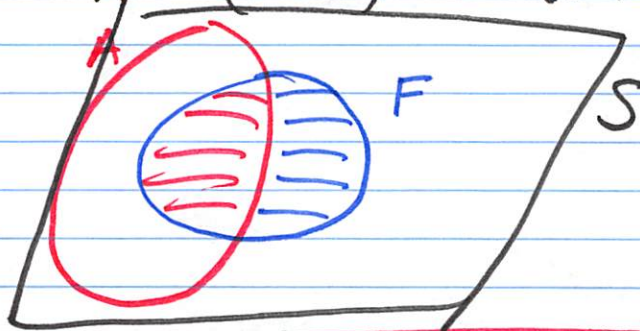
Let's suppose there's another event A.

$$\Pr(F) = \Pr(A) \Pr(F|A) + \Pr(A') \Pr(F|A')$$

Why?

$$\Pr(F) = \Pr(F \cap A) + \Pr(F \cap A')$$

(Note: $F = (F \cap A) \cup (F \cap A')$)



$$\textcircled{2} \quad \Pr(F) = \Pr(F|A) \Pr(A) + \Pr(F|A') \Pr(A')$$

$$= \Pr(F \cap A) + \Pr(F \cap A')$$

① and ② follow from previous material.

11/13

Today: Bayes' Theorem (Ch 9.7)

Recall: If E, F are two events,

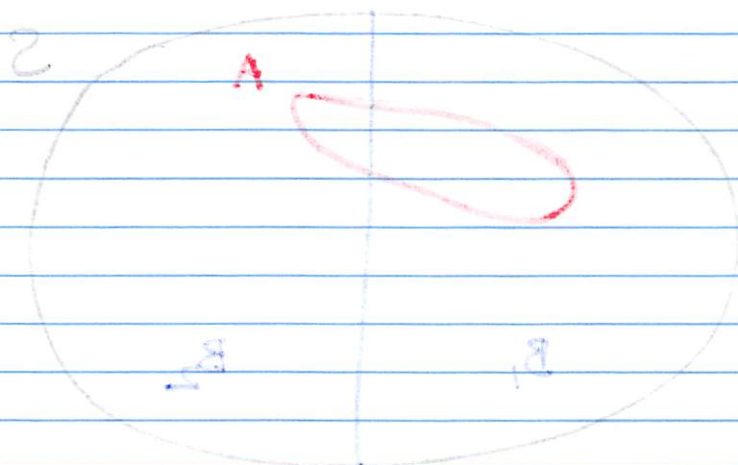
$$P(E|F) = \frac{P(E \cap F)}{P(F)} \Rightarrow P(E \cap F) = P(E|F) \cdot P(F)$$

$$P(F|E) = \frac{P(E \cap F)}{P(E)} \Rightarrow P(E \cap F) = P(F|E) \cdot P(E)$$

Let's suppose A is an event in Ω

and B_1, B_2 are mutually exclusive events

such that $B_1 \cup B_2 = \Omega$.



$$P(B_1 \cup A) = P(B_1|A)P(A)$$

$$= P(A)P(B_1|A)$$

$$P(B_1|A) = \frac{P(B_1 \cap A)}{P(A)}$$

Proposition on the right. Write we don't know that.

Today: Bayes' Thm (Ch 6.7)

11/4/13

Recall If E, F are two events,

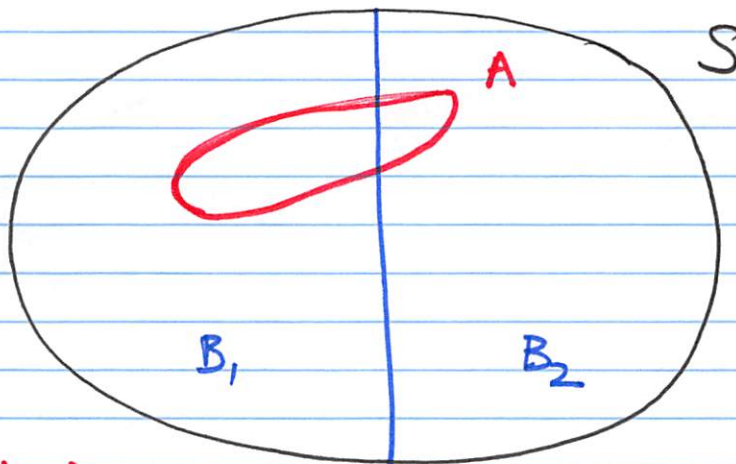
$$\Pr(E|F) = \frac{\Pr(E \cap F)}{\Pr(F)} \Rightarrow \Pr(E \cap F) = \Pr(E|F) \cdot \Pr(F)$$
$$\Pr(F|E) = \frac{\Pr(E \cap F)}{\Pr(E)} \Rightarrow \Pr(E \cap F) = \Pr(F|E) \cdot \Pr(E)$$

Bayes' is sort of a generalization...

Let's suppose A is an event in S

and B_1, B_2 are mutually exclusive events

such that $B_1 \cup B_2 = S$.



$$\Pr(B_1 \cap A) = \Pr(B_1|A) \Pr(A)$$
$$= \Pr(A|B_1) \Pr(B_1)$$

$$\Pr(B_1|A) = \frac{\Pr(B_1 \cap A)}{\Pr(A)}$$

what if we don't know these probabilities on the right?