Nov 20, 2013

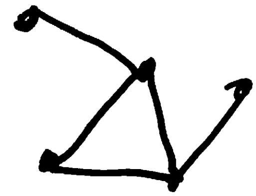
Bring red pen on Monday!

I'll likely post the exam & tell you which parts will be retested.

INFO TO COME TONIGHT!

GRAPHS

Set of vertices and edges.

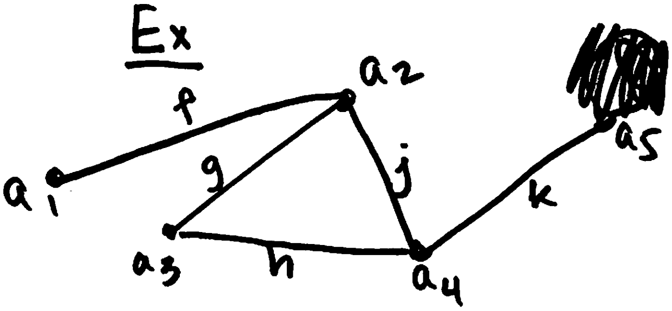


More precisely, $V = \{\text{points}\}$

$$E = \{ \{a,b\} \mid a, b \in V \}$$

($\in$: element)

Ex



$V = \{a_1, a_2, a_3, a_4, a_5\}$

$E = \{ \{a_1, a_2\}, \{a_2, a_3\}, \{a_3, a_4\}, \{a_2, a_4\}, \{a_4, a_5\} \}$

$= \{f, g, h, j, k\}$

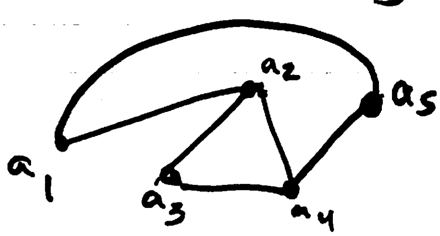
Not an edge:

$\{a_1, a_5\}$

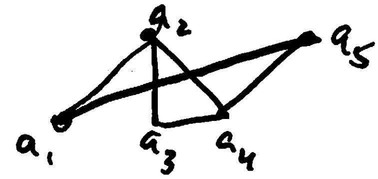
$\{a_2, a_5\}$

$\vdots$

Note We only care about edges & vertices.



is the same graph as



From the book:

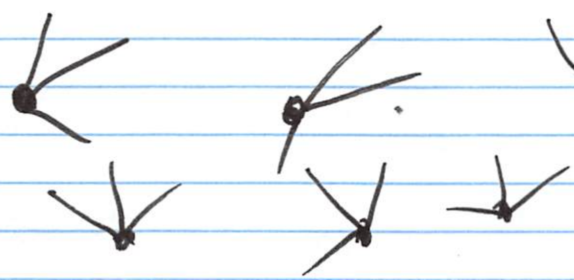
How many vertices will a graph have if

(a) It has 18 edges and each vertex is degree 3?

(b) It has 18 edges, 4 vertices of degree 4 and the others of degree 5?

(a) Let x = # of vertices the graph has.

The sum of the degrees is



$3x$ = Sum of degrees

(sum up 3, x times)

$3x = 2(18)$

$\Rightarrow \boxed{x = 12}$

(the sum of the degrees is twice the # of edges).

(b) Let x = # of vertices

$x-4$ = # of vertices of degree 5.

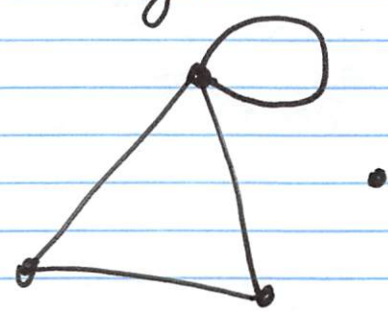
Sum of the degrees: $4 \cdot 4 + (x-4)\cdot 5 = \underset{\text{twice \# of edges}}{2(18)}$

$= 16 + 5x - 20 = 36$

$\Rightarrow -4 + 5x = 36$

$5x = 40 \Rightarrow \boxed{x = 8}$

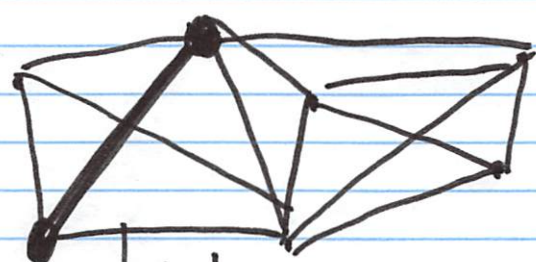
You can have edges from a vertex to itself:



Isolated vertex

The degree of an vertex is the # of edges coming out of it.

If I have n vertices. How many edges that are between two distinct vertices (no) are ~~the~~ possible?

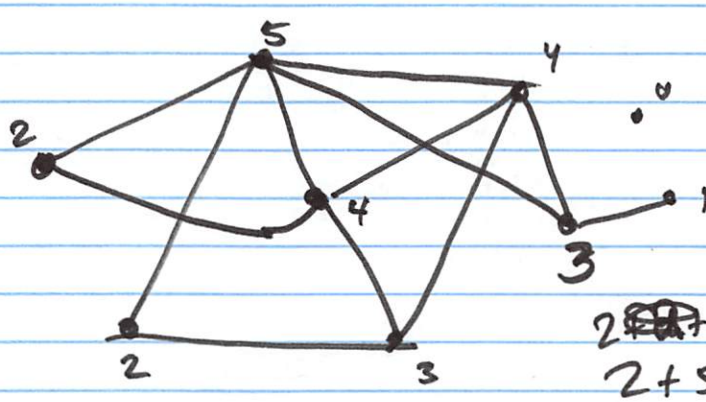


lots!

10 edges

7 vertices $\rightarrow$ $\binom{7}{2}$ possible edges. (Each edge is a choice of 2 vertices among 7)

Suppose we a graph: degree:



2

$2+5+4+0+1+3$
$+3+4+2$

$=$

Thm The sum of the degrees of the vertices of a graph G is twice the # of edges of G. (and therefore it's always even).

Why? Each edge is counted in the degree of each of its two vertices.

Thm In any graph G, the # of vertices with odd degree is even.

Reason: the sum of all the degrees is even. Therefore the sum of the odd degree vertices is even. Therefore there must be an even # of them.

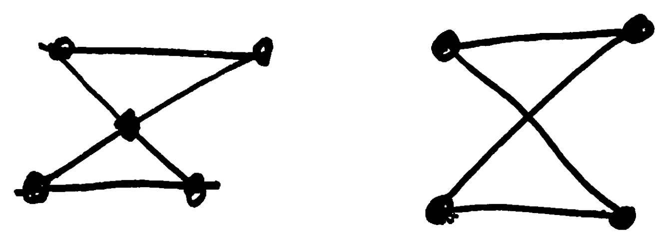
Ex Can I construct a graph with 10 vertices and degrees (3, 3, 3, 3, 3, 3, 3, 3, 3, 6)?

Not possible because there are an odd # of vertices with odd degree.

Graphs are equivalent if they have the same vertex and edge set, regardless of how they "look" in the plane:

we did an example before.

Another example:

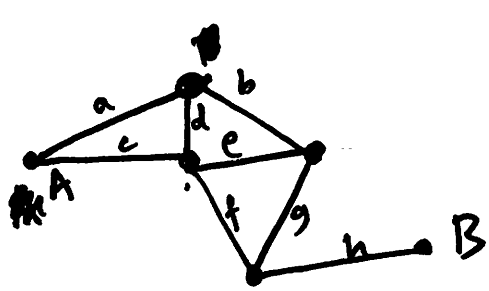


NOT EQUIVALENT
NOT EVEN the same # of vertices.

If they are equivalent:

(1) same # of vertices
(2) Same # of edges
(3) Same # of vertices of each degree.
(4) You can pair up the vertices of the same degree so that if there's an edge joining a pair in 1 graph, there's an edge joining the corresponding pair in the other graph.

Def A path in a graph is a sequence of edges that link two vertices:



Paths from A to B
c e g h
a d f h
etc.

COUNTING # OF PATHS IS TRICKY.