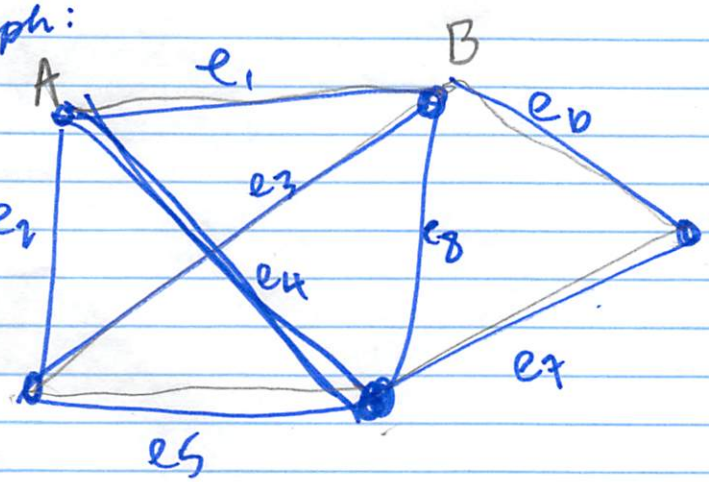
Given a graph:



edges labelled

Path is a sequence of edges that links together two vertices.

ex $e_1 e_3 e_5 e_7 e_6$

is a path from A to B

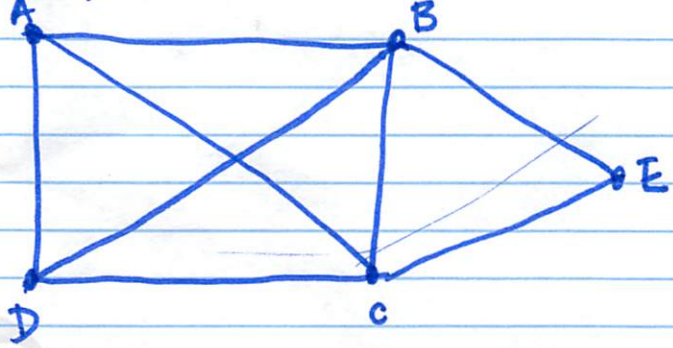
The length of this path is the # of edges used in the path.

Has length 5. (Repeated edges count as many times as repeated)

We limit our discussion to graphs with

- no loops (no paths from a vertex to itself).
- no "parallel edges" Between any two vertices there is at most one edge.

Same picture, different notation (label vertices)



Vertex sequence

ABDCEB

Same as path above.

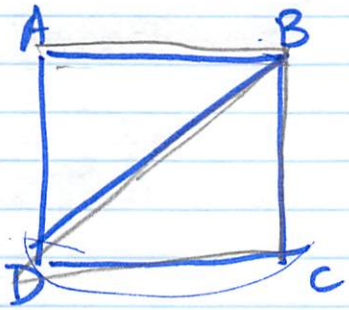
Simple path: one in which no vertex is repeated.

Closed path: path from a vertex to itself.

→ its 1st and last vertices are the same

Circuit Closed path with no repeated edges.

First example has no repeated edges, but it's not a circuit b/c it's not a closed path.

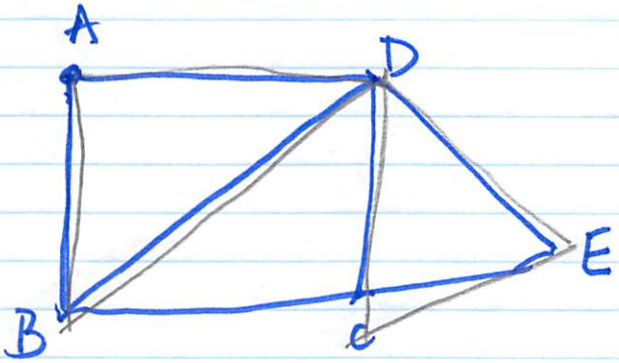


length of path 5

ABCDBA

is a closed path

Not a circuit repeated the edge {A,B}.



length of path 6

ABDCEDA

closed path ✓

circuit ✓

simple circuit ✗

↑ went through D twice.

has no repeated vertices except 1st & last

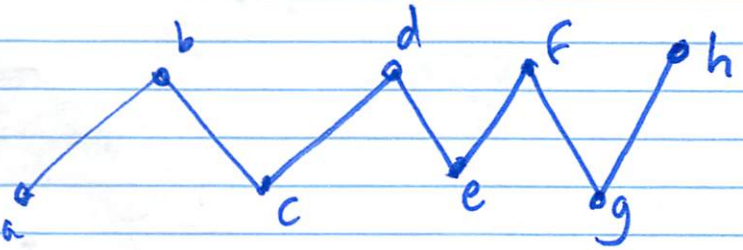
A graph is acyclic if it contains no circuits.

Given a connected graph, a simple path that goes through every vertex of G and has min # of edges, we say this path is a path of minimal length.

To get min length paths, avoid circuits!

(simple paths do not repeat vertex).

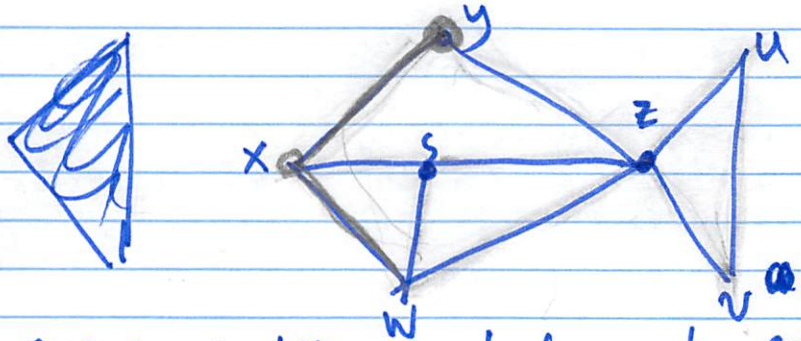
Ex



minimal length path

touches each vertex exactly once. abcdefgh

Minimal # of edges required to do this.



vuzwsxy

Uses every vertex, but not every edge.

Euler path A path in which every edge is used exactly once

Euler circuit An ~~circuit~~ Euler path that starts and ends at the same vertex.

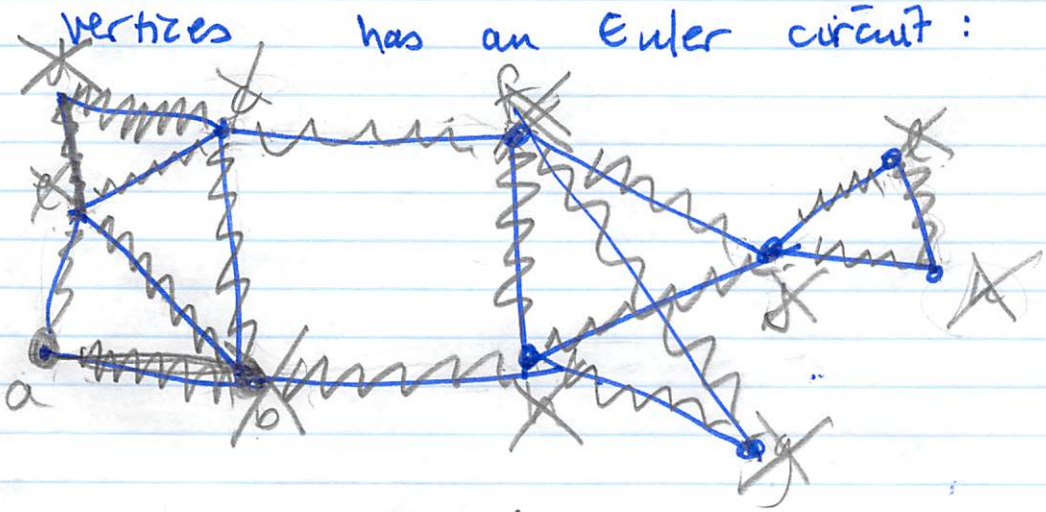
Are there always Eulerian circuits?

NO a graph with some odd-degree edges has no Eulerian circuit.

Thm Let G be a graph. It has an Euler circuit if and only if

1. G is connected.
2. every vertex of G has even degree.

Ex This implies every connected G with even degree vertices has an Euler circuit:



a b e d c b h g f h j l k j f c e a

Fleury algorithm for Finding an Euler circuit (on a connected graph whose vertices are all even degree).

Thm **Euler path** Let G be a graph with 3 or more vertices. Then G contains an Euler path, if and only if (but not a circuit)

(1) G is connected
(2) Two vertices are of odd degree
(3) Every other vertex has even degree.