

$p, q$  statements

$p \vee q$  "  $p$  or  $q$  "

$p \wedge q$  "  $p$  and  $q$  "

$\sim p$  " not  $p$  "

" It is not the case that  $p$  . "

Truth table : keeps track of possible values of everything in the table for any propositions listed.

$\rightarrow$  "implies"

$p \rightarrow q$  "  $p$  implies  $q$  "  
" If  $p$ , then  $q$  "

Ex

If it rains today, I will drive to work.

$p$  = " It rains today. "

$q$  = " I'll drive to work. "

is written  $p \rightarrow q$  .

P	q	$P \rightarrow q$	$q \rightarrow P$
T	T	T	T
T	F	F	T
F	F	T	T
F	T	T	F

$\oplus$  "exclusive or"

$P \oplus q$  p or q but not both.

Use a truth table to find the truth values of  ~~$(p \vee q) \vee p$~~

P	q	$\sim q$	$P \wedge \sim q$	$\sim(P \wedge \sim q)$	$\sim(P \wedge \sim q) \vee P$
T	T	F	F	T	T
T	F	T	T	F	T
F	T	F	F	T	T
F	F	T	F	T	T

This is always true, so it's called a tautology

Def'n — A tautology is a statement with logical value TRUE independent of the truth value of individual statement variables it contains.

• Similarly, a contradiction is a statement that is always false.

$p \wedge (\sim p)$  is a contradiction.

Pf

$p$	$\sim p$	$p \wedge (\sim p)$
T	F	F
F	T	F

19. p. 13

Truth table for  $p \oplus (q \vee r)$

$p$	$q$	$r$	$q \vee r$	$p \oplus (q \vee r)$
T	T	T	T	F
T	T	F	T	F
T	F	T	T	F
T	F	F	F	T
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	T

$P \oplus q$  means  $(p \vee q) \wedge \sim(p \wedge q)$

P	q	$P \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

29 p. 13 Compare truth tables  $\sim p \vee \sim q$   
and  $\sim(p \wedge q)$

[ Put both in a single truth table and compare ]

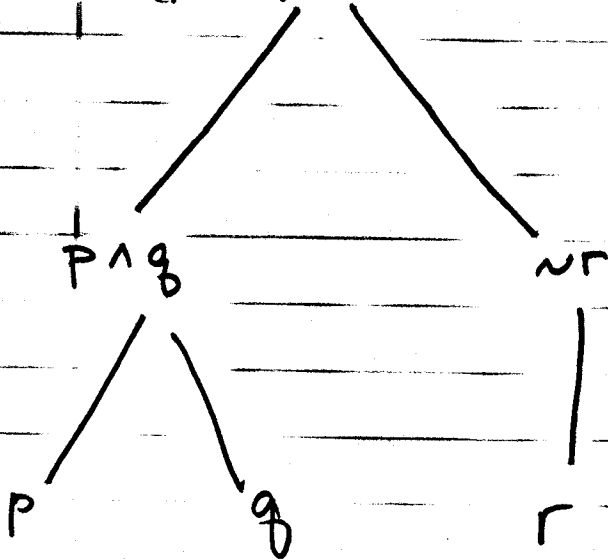
P	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \vee \sim q$	$\sim(p \wedge q)$
T	<del>T</del> T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
<del>F</del> F	<del>F</del> F	T	T	F	T	T

They have same truth values.

These statements are logically equivalent.

# Trees

$$(P \wedge q) \vee \sim r$$

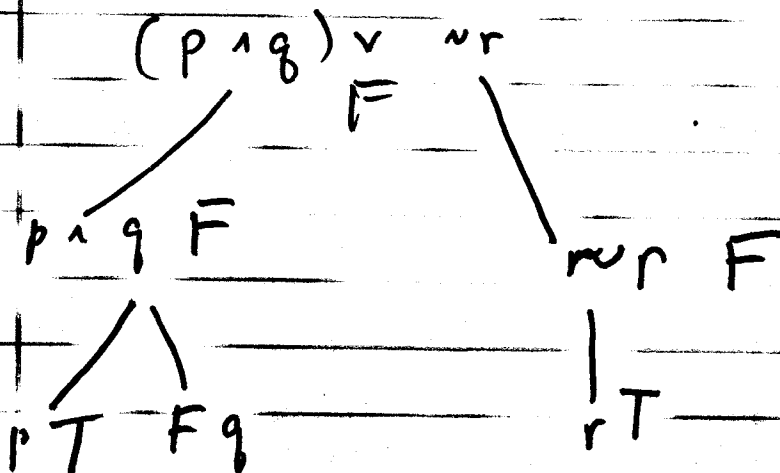


break into smaller units

from which I can determine truth/falschood of the "parent" in tree.

Why is this good? A table requires evaluating the answer for every truth value of P, q, r. A tree is a fast way to get an answer for one value of P, q, r.

Ex Find the value of  $(P \wedge q) \vee \sim r$  when P is T, q is F, r is T.



$P$  = "Jack ~~was~~ is drunk."

$Q$  = "Jack has an accident"

$P \rightarrow Q$

"If Jack is drunk,  
he has an accident."

$Q \rightarrow P$

"If ~~he~~ Jack has an accident  
then ~~he~~ <sup>Jack's</sup> drunk."

~~$P \rightarrow Q$~~

~~is saying P is a success~~

→ To have an accident, it is sufficient  
that he was drunk.

In our story, being drunk guarantees an  
accident on the ice patch.

But maybe if not drunk, you'd  
still have an accident.

It is not necessary that he be drunk.

"Sufficient but not necessary."

But... passing 112 is necessary to  
graduate w/ IT major, but not sufficient.