

Lecture 2

Tuesday, September 3, 2013
12:56 PM

Recap of P and Q Statements:

$P \vee Q$ = "P or Q"

$P \wedge Q$ = "P and Q"

$\sim P$ = "Not P"
"It is not the case that P."

Truth Table: Keeps track of possible values of everything in the table for any propositions listed.

\rightarrow = "implies"

$P \rightarrow Q$ = "P implies Q."
"If P, then Q."

Ex: If it rains today, I will drive to work.

Split into two statements, it would read:

P = "It rains today."
Q = "I'll drive to work."

Is written as $P \rightarrow Q$

Truth Table for Statement:

| P | Q | $P \rightarrow Q$ | $Q \rightarrow P$ |
|---|---|-------------------|-------------------|
| T | T | T | T |
| T | F | F | T |
| F | F | T | T |
| F | T | T | F |

\oplus = "Exclusive Or" or "XOR"

*This is what is commonly thought of when we think of everyday language or statements - either one or the other, but not both.

Lecture Problem:

Use a Truth Table to find the Truth values of: $\sim(P \wedge \sim Q) \vee P$

| P | Q | $\sim Q$ | $P \wedge \sim Q$ | $\sim(P \wedge \sim Q)$ | $\sim(P \wedge \sim Q) \vee P$ |
|---|---|----------|-------------------|-------------------------|--------------------------------|
| T | T | F | F | T | T |
| T | F | T | T | F | T |
| F | T | F | F | T | T |

| | | | | | |
|---|---|---|---|---|---|
| T | F | T | T | F | T |
| F | T | F | F | T | T |
| F | F | T | F | T | T |

This is known as a Tautology.

Definition: Tautology is a statement with logical value true independent of the truth value of the individual statement variables it contains.

Definition: Contradiction is a statement that is always false.

$P \wedge (\sim P)$ is a contradiction. We can see this in a truth table.

| | | |
|---|----------|---------------------|
| P | $\sim P$ | $P \wedge (\sim P)$ |
| T | F | F |
| F | T | F |

Homework Problem (12.2, pg 13, # 19)

Truth table for $P \text{ XOR } (Q \vee R)$

| P | Q | R | $Q \vee R$ | $P \text{ XOR } (Q \vee R)$ |
|---|---|---|------------|-----------------------------|
| T | T | T | T | F |
| T | T | F | T | F |
| T | F | T | T | F |
| T | F | F | F | T |
| F | T | T | T | T |
| F | T | F | T | T |
| F | F | T | T | T |
| F | F | F | F | F |

$P \text{ XOR } Q$ also can be written as: $(P \vee Q) \wedge \sim (P \wedge Q)$

Homework Problem (12.2, pg13, # 29)

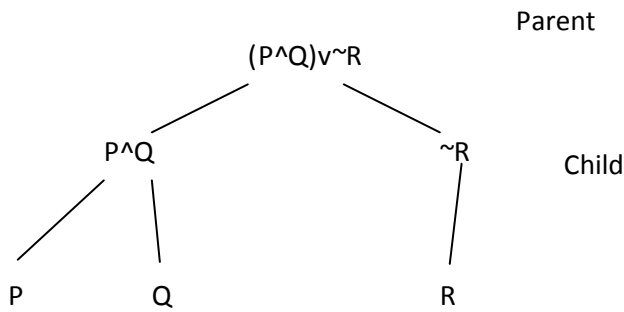
Compare truth table $\sim P \vee \sim Q$ and $\sim (P \wedge Q)$

*put both statements in a truth table

| P | Q | $\sim P$ | $\sim Q$ | $P \wedge Q$ | $\sim P \vee \sim Q$ | $\sim (P \wedge Q)$ |
|---|---|----------|----------|--------------|----------------------|---------------------|
| T | T | F | F | T | F | F |
| T | F | F | T | F | T | T |
| F | T | T | F | F | T | T |
| F | F | T | T | F | T | T |

They are the same and logically equivalent, as they have the same truth values.

Trees

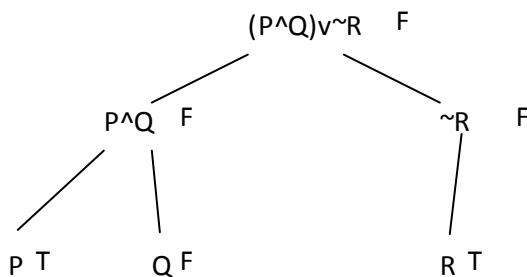


Why are trees good?

A table requires evaluating the answer for every Truth Value of P, Q, R. A Tree is a fast way to get an answer for one value of P, Q, R.

Example:

Find value of $P^Qv~R$ when P is T, Q is F, and R is T.



Causality

Guy drives down road and kills someone. There are two potential cases, which caused it.

Note: Causality is tricky - can't assume causality.

Difference between necessary and sufficient conditions.

Ice patch vs. Drinking

P = "Jack is drunk."

Q="Jack has an accident."

| | | |
|-------------------|---|--|
| $P \rightarrow Q$ | = | If Jack is drunk, then Jack has an accident. |
| $Q \rightarrow P$ | = | If Jack has an accident, then Jack is drunk. |

$P \rightarrow Q$ - To have an accident it is sufficient he was drunk. In our story, being drunk guaranteed the accident on an ice patch, but maybe you'd still have an accident anyway. It is not necessary he be drunk.

Another example statement...

Passing 112 is necessary to graduate with an IT Major but is not sufficient.