MATH Practice 1 (for Exam 1)

Professor Goldin

- 1. Determine whether the following are statements. Write "STATEMENT" or "NOT STATE-MENT" for each item.
 - (a) Janis Joplin is cool and Justin Bieber is a dork.
 - (b) The final for Physics 104 at GMU is hard.
 - (c) Granite rock is harder than limestone.
 - (d) This sentence is FALSE.
 - (e) If pigs fall from the sky, Dr. Goldin will give each student in Math 112 some money.
- 2. Construct a truth table for the statement $(q \lor p) \to (p \land q) \lor \neg q$.
- 3. Consider the following statements:

p =Jerry Jones has won the lottery.

q = Jerry Jones' wife filed for divorce.

Write the English words for the symbolic statement.

- (a) $p \rightarrow q$
- (b) $(p \land \neg q) \lor [(\sim p) \land q]$.
- (c) Write the symbolic statement for the English words, "If Jerry's wife did not file for divorce, then Jerry has won the lottery."
- (d) Write the symbolic statement for the English words, "Jerry has won the lottery only if his wife filed for divorce."
- 4. Prove the logical equivalence

$$p \rightarrow \sim q \Leftrightarrow \sim p \lor \sim q$$

- 5. Consider the statement, "You will get a million dollars in royalties if your book is a best-seller."
 - (a) Write the contrapositive statement.

- (b) Write the converse statement.
- 6. Given that p is true and q, r are false, determine whether the following is TRUE, FALSE, or UNDETERMINED.

$$(\mathbf{p} \wedge \sim \mathbf{q}) \vee \mathbf{r}.$$

7. Consider the universal set $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ and the open sentence

$$p(x) = 5x + 2 < 25$$

Answer true or false.

- (a) p(3). (b) $p(2) \land p(5)$. (c) $\forall x p(x)$. (d) $\exists x p(x)$.
- (e) $p(6) \to p(1)$.
- 8. Use the laws of logical equivalences to negate the following statement: $(\sim p \lor q) \land q$
- 9. Show the law of modus ponens holds:

$$\mathbf{p} \land (\mathbf{p} \to \mathbf{q}) \Rightarrow \mathbf{q}$$

10. Write a convincing argument that

$$(\sim [\forall x p(x)]) \Leftrightarrow (\exists x \sim p(x)).$$

11. Let $U = \{all \text{ people in the United States.}\}$ Consider the statement

p(x, y) = "x is the mother of y."

Evaluate the truth value.

- (a) $\forall y \exists xp(x,y)$
- (b) $\exists x \forall yp(x,y)$.
- 12. Write an equivalent statement below without using the symbol \rightarrow . Simplify your answer at the end so it has no parentheses.

 $q \rightarrow p \wedge \sim r$.

13. True or False.

- (a) A predicate is an example of a statement.
- (b) A syllogism is a logical conclusion following from two or more other statements.
- (c) The following is an example of a syllogism:

All students learn something. All people enrolled in Math 112 are students. Conclusion: All people enrolled in Math 112 learn something.

(d) $\sim (p \land q) \Leftrightarrow \sim p \land \sim q$