MATH 112-002 Final Exam

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Be sure to show all work on these pages. You will not get credit if your work is not shown, even if you have the correct answer. By taking this exam, you agree to follow the GMU honor code, which is to do your own work, and to keep your work covered to help others do their own work.

Honor Code Signature. I understand that cheating compromises the integrity of the course for everyone, including for students not directly involved. By signing below, I am stating the work included in this document is entirely my own and no one else's. I will not share my work with anyone else during the exam.

Signature: ____

1. *(6 points)* Make the required statement.

   (a) Negate the statement, "I will drive or I will walk". Simplify to make it sound like something you would say!

   (b) Find the contrapositive of the statement "If I can't have chocolate, then you can't have chocolate."

2. *(6 points)* Find the truth value of the following statements, when p is T, q is F, and r is T. Justify your answer.

   (a) $\sim (p \rightarrow q) \vee (\sim p \wedge \sim r)$

   (b) $(r \leftrightarrow p) \rightarrow \sim (q \wedge \sim p)$.

3. *(6 points)* Evaluate the truth value of the statements. Justify your answer.

   (a) $\exists x\ [(x < 0) \rightarrow (3x^2 < 0)]$

   (b) $\forall x\ [(x - 6 \geq 2) \vee x < 1]$

4. *(6 points)* Is the statement $\sim (p \vee q)$ logically equivalent to $p \rightarrow \sim q$? Prove your answer.

5. *(8 points)* Consider the statements below:

   I. If I overeat, I have a stomach ache, and I have a stomach ache.

   II. I did not overeat, or I do not have a stomach ache.

   (a) Translate the sentences into symbols, using

   $$p = \text{"I overeat."} \quad q = \text{"I have a stomach ache."}$$

   I. ____

   II. ____

   (b) Use a truth table to determine whether the two statements are logically equivalent. State your conclusion.

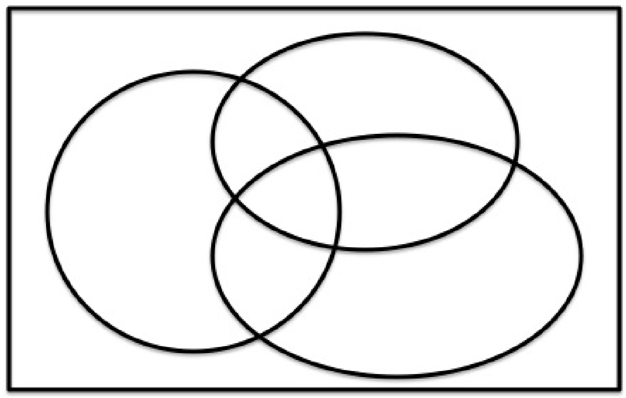
6. *(6 points)* Let A, B, C be sets. Draw a Venn diagram illustrating the following expression: $(A' \cap B) \cup C$.

7. *(12 points)* The results of a a survey of GMU students resulted in the following information:

120 students love the color blue.	70 love the colors blue and red.
145 love the color red.	80 love red and green.
135 love the color green.	60 love blue and green.

15 love blue, red, and green.

(a) Draw the Venn diagram for the results of the study, and label each region by the number of elements in that region.



(b) How many people surveyed love exactly one color?

(c) How many people love blue but not red?

8. *(8 points)* A company is issuing a security code for employees to enter the building. Each code is two letters followed by 6 digits.

(a) How many codes are possible?

(b) If a code is selected at random, what is the probability that the first three digits are 7s?

9. *(8 points)* There are eight students in a class, {John, Jack, Justin, Joshua, Karen, Kerrie, Kayla, and Karina}.

   (a) In how many ways can the students be arranged in a row?

   (b) In how many of these ways will the students {John, Jack, Justin, Joshua} be next to each other?

10. *(8 points)* An international conference consists of 21 people from France, 31 people from Japan, and 11 people from Canada.

   (a) In how many ways could one make an international committee of 9 people with 3 people from each country?

   (b) If a group of 9 people are chosen randomly, what is the probability that it will contain at least one person from Canada?

11. *(8 points)* A sample of a 5 patient records are chosen at random from a file of 60 patient records. Suppose 7 patients in the file are known to have blood type O negative.

   (a) Find the probability that the sample contains at most one patient with O negative blood type.

   (b) Find the probability that the sample contains more than 3 patients with O negative blood type.

12. *(8 points)* A fish company tests its fish for mercury before selling it. The test will indicate whether the fish has an unacceptable level of mercury for human consumption. Assume that, among fish with high levels of mercury in it, the test says the fish is unacceptable in 85 percent of the cases. Among fish with acceptable mercury levels, the test says it is acceptable 90 percent of the time. In fact, the fish being tested is contaminated with high levels of mercury approximately 7 percent of the time.

    (a) Make a tree indicating the probabilities of whether the fish have high levels of mercury and the test outcome. **Label the edges and the vertices of the tree by the appropriate probabilities and outcomes.**

    (b) Find the probability that a fish chosen at random is *not* contaminated by mercury but the test says it is unacceptable due to high mercury levels.

    (c) What is the probability that a fish chosen at random has a tests saying it is unacceptable?

    (d) A thousand fish tested to have normal levels of mercury levels. Among these fish, how many do we expect were actually inappropriate for human consumption?

13. *(5 points)* Assume that the probability of a person being born in any given month is exactly 1/12. Five people are chosen randomly on the streets of New York City. What is the probability that at least two of them were born in the same month?

14. *(4 points)* In the expansion of $(x+y)^{26}$, find the coefficient of the term x^7y^{19} and write your result as an integer.

15. *(9 points)* Below are the survey results regarding the majors of members of two different clubs at GMU. The clubs meet at the same time, so no students are enrolled in both clubs. A

Table 1: Club Membership and GMU Major

	Biology	Information Technology	Psychology
Cooking Club	700	600	100
Hiking Club	150	230	350

student is chosen randomly from the participants.

(a) Find the probably that the student is in the Cooking Club *or* is an Information Technology major.

(b) Given that the student is in the Hiking club, find the probability that he or she is a Psychology major.

(c) Are the events "Member of the Cooking Club" and "Biology major" independent? Show a calculation to support your answer.

16. *(7 points)* True or False.

T or F For any graph, the sum of the degrees of the vertices of G is twice the number of edges of G.

T or F A simple path is a path in which no edge is repeated.

T or F A circuit may touch many vertices, but it can touch each edge at most once.

T or F A graph is planar if and only if no matter how it is drawn in the plane, no edges are touching except at vertices.

T or F A loop from a vertex to itself does *not* count as an edge.

T or F There exists a graph having 30 vertices of degree two, and three vertices of degree one.

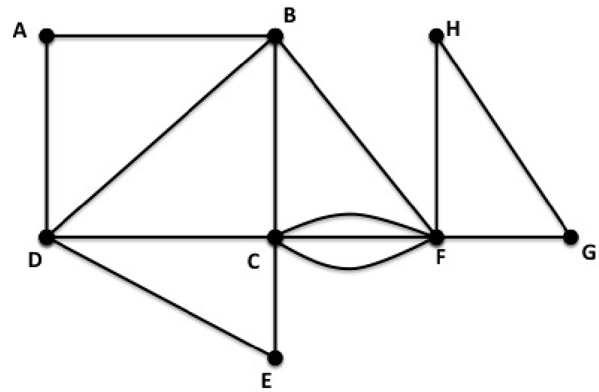
T or F An Euler path must include every edge, but it might include some edges more than once.

17. *(6 points)* Justify the answer with a theorem or a principle. G is a connected simple graph with no loops. It has 9 vertices and 31 edges. Does G have a Hamiltonian circuit?

18. *(6 points)* Consider a connected graph with 20 vertices, labeled $v_1, v_2, \ldots, v_{20}$. The degrees at each vertex are listed here: $(2, 4, 5, 6, 8, 10, 18, 22, 30, 4, 6, 9, 12, 8, 2, 12, 14, 20, 22, 24)$. In other words, the degree of v_1 is 2, the degree of v_2 is 4, the degree of v_3 is 5, etc.

Does this graph have an Euler path, or is it not determined by the information? (YES, NO, UNDETERMINED). Justify your response with a theorem or principle.

19. *(8 points)* Consider the graph below. Suppose we start at point B to create an Euler circuit,



and we begin with the path BCFCF. What are *all* the possibles next edges to add to this path following Fleury's algorithm to find an Euler circuit? Specify the edges by circling all possible choices for the next edge according to the algorithm.

20. *(10 points)* Answer the following questions about the graphs below.

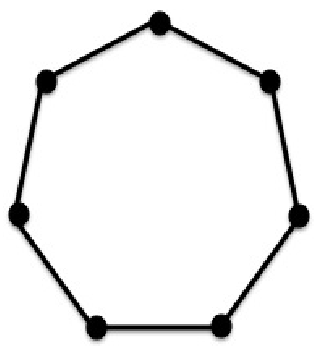


Figure 1: Graph for Problem 20(a)

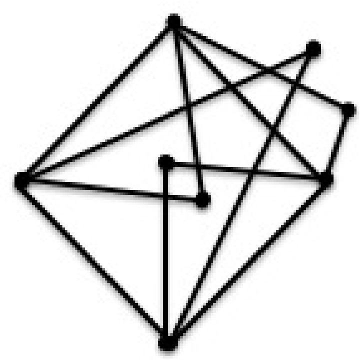
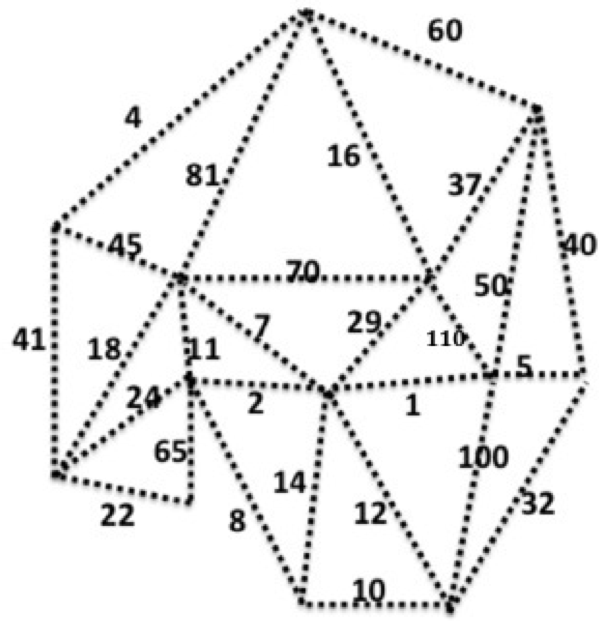


Figure 2: Graph for Problem 20(b)

(a) Find the chromatic number of the graph on the left (Figure 1). Justify your answer.

(b) Is the graph on the right (Figure 2) a planar graph? Justify your answer.

21. *(6 points)* Find a minimal spanning tree of the weighted graph below. Indicate your tree by drawing a solid line over the dotted edges.



22. *(6 points)* Draw a graph with the adjacency matrix given by $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 0 & 3 & 2 \\ 3 & 3 & 0 & 1 \\ 4 & 2 & 1 & 2 \end{bmatrix}$. **Label the vertices of your graph** v_1, v_2, v_3, v_4.

23. *(6 points)* Find the number of length two paths from v_1 to v_3 in the graph with adjacency matrix given by $A = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ using matrix multiplication. Be sure to show your work.

24. Extra Credit. *(5 points)* Find a Hamiltonian circuit on the graph below.