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A nonconforming finite element method for fluid–structure interaction problems

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Abstract

In this paper, we develop a nonconforming finite element methodology using a *three-field* formulation to analyze a fluid–structure interaction problem. The methodology is used to couple a Lagrangian model describing the structure with the arbitrary Lagrangian–Eulerian strategy used to describe the fluid in order to simulate a full unsteady physical phenomenon. Consistency error estimates are obtained which show that the numerical scheme employed yields a first order approximation for the solution to the fluid–structure interaction problem. Finally, we present a discrete energy estimate to demonstrate the stablity of the proposed method.

Keywords: Fluid–structure interaction; Three-field; Finite elements; Nonconforming; Arbitrary Lagrangian–Eulerian

1. Introduction

An efficient solution to a fluid–structure interaction problems is still a challenging one in computational mathematics. Direct numerical solution of the highly nonlinear equations governing even the most simplified two-dimensional models of fluid–structure interaction requires that both the flow field and the domain shape be determined as part of the solution, since neither is known a priori. Previous algorithms have decoupled the solid and the fluid mechanics, solving for each separately and converging iteratively to a solution which satisfies both. However, in order to predict the dynamic response of a rigid or flexible structure in a fluid flow, ideally the equations of motion of the structure and the fluid should be solved simultaneously.

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While solving the fluid–structure interaction problem as a coupled problem, it is important to realize several key facts:

- The fluid produces tractions that deform the structure. These deformations alter the flow field and hence, result in modified fluid tractions.
- The structural equations are usually formulated with material (Lagrangian) coordinates, while the fluid equations are typically written using spatial (Eulerian) coordinates.
- The nodes on the fluid mesh are attached to the surface of the structure and have to move as the structure deforms.

Hence, the solution of the coupled fluid–structure dynamic equations can become very complicated and also can lead to severe mesh distortions when the structure undergoes large deformation. A variety of approaches have been developed to solve fluid–structure interaction problems, including the co-rotational approach [11,16], dynamic meshes [2], parallel methods [15] and the arbitrary Lagrangian–Eulerian formulation [9]. A more general overview of numerical methods to study fluid–structure interaction problems can be found in [17].

To support a flexible meshing procedure for a fluid–structure interaction problem, it is crucial that an efficient method be employed to join the fluid and structure sub-meshes together, even though the finite element nodes of the fluid and structure at the common interface may not, in general, be coincident. To accomplish this one may employ a Lagrange multiplier to take care of the continuity constraints, i.e.,

$$u_S - u_F = 0 \quad \text{on} \quad \Gamma_{SF},$$

where $\Gamma_{SF} = \partial\Omega_S \cap \partial\Omega_F$ is the interface between the fluid domain Ω_F and structure domain Ω_S and u_S, u_F are the values of the test or trial function u on Γ_{SF} from the two sides. Here u_F may represent the velocity of the fluid and u_S the time derivative of the displacement of the structure. With such a technique, the above equation is enforced only weakly, with the jumps $u_S - u_F$ being made orthogonal to a space of Lagrange multipliers on Γ_{SF}. (An alternative method, not involving Lagrange multipliers, could be based on *hanging nodes*, see, e.g., [8,18].) The *mortar finite element method* (see, e.g., [3–5,10,19,21,22,24] and the references therein) is one example of a Lagrange multiplier technique. These methods are becoming increasingly popular as specialized domain decomposition techniques for treating second-order partial differential equations on any type of domain, with very few restrictions on the grid related to the discretization procedure. One can also employ much more general *three-field* methods, where one also has a third field z on the interface. This variable corresponds to the exact solution on Γ_{SF}, and one now introduces two Lagrange multipliers to deal with the constraints

$$u_S - z = 0, \quad u_F - z = 0 \quad \text{on} \quad \Gamma_{SF}.$$

See, e.g., [1,6,20] for variants of this idea.

The purpose of our paper is three-fold. First, we present a nonconforming finite element formulation for a fluid–structure interaction model problem using a three-field approach. Secondly, we prove a consistency result satisfied by the finite element solution for the numerical scheme presented. Finally, we present a discrete energy estimate for the proposed method which confirms the stability of our scheme.

2. Model problem

Let $\mathbf{x} = (x_1, x_2) \in \mathbb{R}^2$. We consider an initial configuration of two rectangular domains $\Omega_F^0 = [0,1] \times [0,1]$ and $\Omega_S^0 = [1,2] \times [0,1]$ coincident at the initial interface $\gamma_0 = \{\mathbf{x} : x_1 = 1, 0 \leqslant x_2 \leqslant 1\}$. Assuming that a viscous incompressible fluid occupies $\Omega_F(t)$ while an elastic structure occupies $\Omega_S(t)$, the interface between the two may move as the system evolves (see Fig. 1). At any instance $t \geqslant 0$, we model the change in fluid velocity $\mathbf{u} = (u_1, u_2) \in \mathbb{R}^2$ and pressure $\hat{p}$ using the Navier–Stokes equations,

$$\sum_{i=1}^{2} \frac{\partial}{\partial x_i}\left[\mu_{\mathrm{F}}\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right)\right] - \frac{\partial \hat{p}}{\partial x_j} = \rho_{\mathrm{F}}\left(\frac{\partial u_j}{\partial t} + \nabla\cdot(u_j\mathbf{u}) - f_j\right), \tag{1}$$

for $j = 1,2$ and $\forall \mathbf{x} \in \Omega_{\mathrm{F}}(t)$. Here, μ_{F} is the dynamic viscosity of the fluid, ρ_{F} is the fluid density, and $\mathbf{f} = (f_1, f_2)$ is the applied force. Additionally, due to the incompressible nature of the fluid, $\nabla\cdot\mathbf{u} = 0$. Applying the incompressibility condition, Eq. (1) becomes for $j = 1,2$,

$$\frac{\partial u_j}{\partial t} - \nu\Delta u_j + \mathbf{u}\cdot\nabla u_j + \frac{\partial p}{\partial x_j} = f_j, \tag{2}$$

$\forall \mathbf{x} \in \Omega_{\mathrm{F}}(t)$, $t \geqslant 0$, where $\nu = \frac{\mu_{\mathrm{F}}}{\rho_{\mathrm{F}}}$ is the kinematic viscosity and $p = \frac{\hat{p}}{\rho_{\mathrm{F}}}$.

Now let $\mathbf{d}$ represent the displacement of the structure from its initial position. Then we can model the change in $\mathbf{d}$ using only the initial solid domain Ω_{S}^0. The structure is modelled via the equations

$$\sum_{i=1}^{2} \frac{\partial}{\partial x_i}\left[\mu_{\mathrm{S}}\left(\frac{\partial d_j}{\partial x_i} + \frac{\partial d_i}{\partial x_j}\right)\right] + \lambda\frac{\partial}{\partial x_j}(\nabla\cdot\mathbf{d}) = \rho_{\mathrm{S}}\left(\frac{\partial^2 d_i}{\partial t^2} - g_j\right) \tag{3}$$

for $j = 1,2$ and $\forall \mathbf{x} \in \Omega_{\mathrm{S}}^0$, where λ and μ_{S} are the Lamé coefficients, ρ_{S} is the solid density, and $\mathbf{g} = (g_1, g_2)$ is the applied load on the structure. Letting $\mu = \mu_{\mathrm{S}}$ and $\varepsilon = \frac{\mu+\lambda}{\rho_{\mathrm{S}}}$, Eq. (3) becomes for $j = 1,2$,

$$\frac{\partial^2 d_j}{\partial t^2} - \mu\Delta d_j - \varepsilon\frac{\partial}{\partial x_j}(\nabla\cdot\mathbf{d}) = g_j, \tag{4}$$

$\forall \mathbf{x} \in \Omega_{\mathrm{S}}^0$. On the left boundary of $\Omega_{\mathrm{F}}(t)$, we assume a no-slip condition. On the upwind boundary $\{\mathbf{x} : 0 \leqslant x_1 \leqslant 1,\ x_2 = 0\}$, we fix the interface γ at its original position and assume zero velocity. Assuming that the velocity on the downwind boundary Γ_{DW} is known and in the downwind direction, the boundary conditions for $\mathbf{u}$ are given by

$$\mathbf{u} = \begin{cases} \mathbf{0}, & \mathbf{x} \in \partial\Omega_{\mathrm{F}}(t)\backslash\{\gamma \cup \Gamma_{\mathrm{DW}}\}, \\ (0, \tilde{u}), & \mathbf{x} \in \Gamma_{\mathrm{DW}}. \end{cases}$$

We also assume homogeneous Dirichlet boundary conditions on $\partial\Omega_{\mathrm{S}}(t)\backslash\gamma$. On the interface, we enforce continuity of the velocities, i.e.,

$$\mathbf{u}|_{\gamma} = \left.\frac{\partial \mathbf{d}}{\partial t}\right|_{x_1=1}, \tag{5}$$

as well as continuity of flux,

$$\boldsymbol{\lambda}_{\mathrm{F}}|_{\gamma} + \boldsymbol{\lambda}_{\mathrm{S}}|_{\gamma_0} = 0, \tag{6}$$

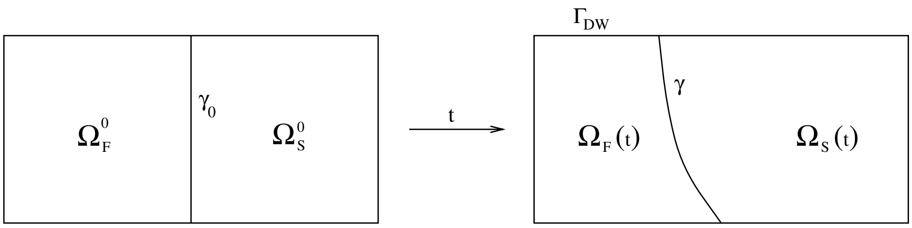

Fig. 1. Deformation of the fluid and solid sub-domains over time.

where $\lambda_{\mathrm{F}}^{(j)} = \nu(\nabla u_j \cdot \mathbf{n}) - pn_j$ is evaluated on γ, the interface, and $\lambda_{\mathrm{S}}^{(j)} = \mu(\nabla d_j \cdot \mathbf{n}) + \varepsilon(\nabla \cdot \mathbf{d})n_j$ is evaluated at the original position of the interface since $\mathbf{d}$ represents the displacement from the initial position, $\mathbf{n} = (n_1, n_2)$ being the appropriate outward normal vector.

In order to account for the changing nature of our fluid domain $\Omega_{\mathrm{F}}(t)$, we wish to define a dynamic mesh when discretizing in space. However, to avoid extreme mesh distortion near the interface, we also choose to move the mesh independently of the fluid velocity on the interior of $\Omega_{\mathrm{F}}(t)$. Such a scheme, called an arbitrary Lagrangian–Eulerian formulation, is commonly applied when studying fluid–structure interaction [9,13,14]. In particular, we allow mesh nodes to move only in the x_1-direction in order to facilitate computations with nonconforming discretizations.

Let x_2 be a fixed value from $\{x_2 : 0 < x_2 < 1\}$. We want the grid velocity of any node along the horizontal line segment at x_2 in $\Omega_{\mathrm{F}}(t)$ to satisfy $w = 0$ whenever $x_1 = 0$ and $w = \dot{\gamma}$ at the interface. If $\gamma(t, x_2)$ represents the x_1-coordinate of the interface, then let

$$w(t,\mathbf{x}) = \frac{x_1}{\gamma(t,x_2)}\dot{\gamma}(t,x_2).$$

The associated characteristic variable must satisfy

$$\frac{\mathrm{d}}{\mathrm{d}t}x_1^{(s)}(t,\xi) = w(t,x_1^{(s)}(t,\xi),x_2),$$
$$x_1^{(s)}(s,\xi) = \xi$$

$\forall\xi \in (0,\gamma(s,x_2))$. Thus,

$$x_1^{(s)}(t,\xi) = \xi\frac{\gamma(t,x_2)}{\gamma(s,x_2)}.$$

Let $\mathbf{v}(t,\xi,x_2) = \mathbf{u}(t,x_1^{(s)}(t,\xi),x_2)$. Then

$$\frac{\partial\mathbf{v}}{\partial t}(t,\xi,x_2) = \frac{\partial\mathbf{u}}{\partial t}(t,x_1^{(s)}(t,\xi),x_2) + w(t,x_1^{(s)}(t,\xi),x_2)\frac{\partial\mathbf{u}}{\partial x_1}(t,x_1^{(s)}(t,\xi),x_2) \tag{7}$$

so Eq. (2) becomes for $j = 1,2$,

$$\left[\frac{\partial v_j}{\partial t} - \nu\Delta u_j + (u_1 - w)\frac{\partial u_j}{\partial x_1} + u_2\frac{\partial u_j}{\partial x_2} + \frac{\partial p}{\partial x_j}\right](t,x_1^{(s)}(t,\xi),x_2) = f_j(t,x_1^{(s)}(t,\xi),x_2). \tag{8}$$

3. A nonconforming finite element method

Choose $\Delta t > 0$ and let $t^n = n\Delta t$, $\phi^n(\mathbf{x}) = \phi(t^n,\mathbf{x})$. We subdivide Ω_{F}^0 and Ω_{S}^0 into triangulations via regular [7] families of meshes. It should be noted that these grids are independent with no compatibility enforced across the interface γ_0, as in Fig. 2.

Assuming a piecewise linear approximation Γ^n for the interface γ at $t = t^n$, we wish to find finite element approximations for $\mathbf{u}^n$, p^n, and $\mathbf{d}^n$, namely $\mathbf{U}^n$, P^n, and $\mathbf{D}^n$, using a weak formulation for the fluid–structure interaction problem. Define Ω_{F}^n to be the approximation of $\Omega_{\mathrm{F}}(t^n)$ using Γ^n. An approximation for the characteristic curve $x_1^{(s)}(t,\xi)$ is, for any fixed x_2, $X_1^n(t) = X_1^n + W^n(X_1^n,x_2)(t-t^n)$, $\forall X_1^n \in (0,\Gamma^n(x_2))$, $t \in [t^n,t^{n+1}]$, where

$$W^n(X_1^n,x_2) = \frac{X_1^n}{\Gamma^n(x_2)}U_1^n(\Gamma^n(x_2),x_2)$$

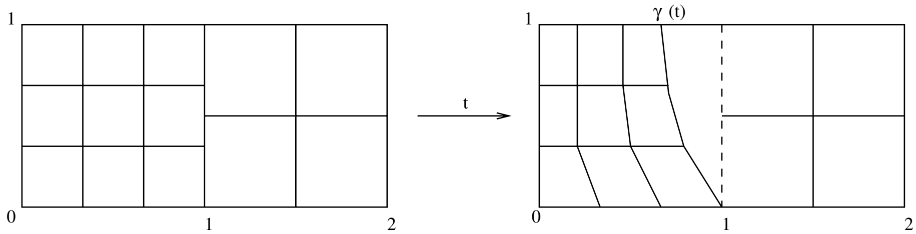

Fig. 2. Evolution of a nonconforming mesh.

and $\Gamma^{n+1}(x_2) = \Gamma^n(x_2) + \Delta t U_1^n(\Gamma^n(x_2), x_2)$. Thus, $X_1^{n+1} = X_1^n + W^n(X_1^n, x_2)\Delta t = X_1^n(t^{n+1})$, which we will use to move the mesh nodes. We then discretize the acceleration by

$$\frac{\partial u_j}{\partial t}(t, x_1^{(s)}(t,\xi), x_2) \approx \frac{1}{\Delta t}(U_j^{n+1}(X_1^{n+1}, x_2) - U_j^n(X_1^n, x_2)).$$

Let $\overline{U}_j^{n+1}(X_1^n, x_2) = U_j^{n+1}(X_1^n + W^n(X_1^n, x_2)\Delta t, x_2) = U_j^{n+1}(X_1^{n+1}, x_2)$. Then a discretization of (8) is given by

$$\frac{1}{\Delta t}(\overline{U}_j^{n+1} - U_j^n) - \nu\Delta\overline{U}_j^{n+1} + (U_1^n - W^n)\partial_{x_1}\overline{U}_j^{n+1} + U_2^n\partial_{x_2}\overline{U}_j^{n+1} + \partial_{x_j}\overline{P}^{n+1} = \bar{f}_j^{n+1}, \tag{9}$$

for $j = 1, 2$, where $\overline{P}$ and $\bar{f}_j$ are defined in the same way as $\overline{U}_j$. Similarly, a discretization of (4) is

$$\frac{1}{(\Delta t)^2}(D_j^{n+1} - 2D_j^n + D_j^{n-1}) - \mu\Delta D_j^{n+1} - \varepsilon\partial_{x_j}(\nabla\cdot\mathbf{D}^{n+1}) = g_j^{n+1} \tag{10}$$

for $j = 1, 2$.

For any connected bounded polygonal domain $\Omega \in \mathbb{R}^2$ let the boundary $\partial\Omega = \overline{\partial\Omega_{\rm D}} \cup \overline{\partial\Omega_{\rm N}}$ (where $\partial\Omega_{\rm D}$ is the Dirichlet boundary and $\partial\Omega_{\rm N} = \partial\Omega\backslash\partial\Omega_{\rm D}$ is the Neumann boundary). Using standard Sobolev space notation, let $H_{\rm D}^1(\Omega) = \{u \in H^1(\Omega) \mid u = 0 \text{ on } \partial\Omega_{\rm D}\}$, where we are using $H^k(\Omega)$ to denote the space of functions with k generalized derivatives on Ω. We set $L_2(\Omega) = H^0(\Omega)$. For any portion of the boundary $\gamma \subset \partial\Omega$, the space $H^{1/2}(\gamma)$ is the set of traces over γ of all the functions of $H^1(\Omega)$ and $H^{-1/2}(\gamma)$ is its topological dual space. For any functional space $X(\Omega)$, the bold symbol $\mathbf{X}(\Omega)$ stands for the product $X(\Omega) \times X(\Omega)$ so that, for instance, $\mathbf{H}_{\rm D}^1(\Omega) = H_{\rm D}^1(\Omega) \times H_{\rm D}^1(\Omega)$.

We then choose finite dimensional subspaces $\mathbf{V}_{\rm F}^n \subset \mathbf{H}_{\rm D}^1(\Omega_{\rm F}^n)$, $W_{\rm F}^n \subset L_0^2(\Omega_{\rm F}^n)$, $\mathbf{V}_{\rm S}^n \subset \mathbf{H}_{\rm D}^1(\Omega_{\rm S}^0)$, $\mathbf{M}_{\rm F}^n \subset \mathbf{H}^{-\frac{1}{2}}(\Gamma^n)$, $\mathbf{M}_{\rm S}^n \subset \mathbf{H}^{-\frac{1}{2}}(\Gamma^0)$ and $\mathbf{Y}^n \subset \mathbf{H}^{\frac{1}{2}}(\Gamma)$, where Γ is an interface space corresponding to the traces of the true solution. Let us now define the global finite element space to be

$$\mathscr{X}^n = \mathbf{V}_{\rm F}^n \times W_{\rm F}^n \times \mathbf{V}_{\rm S}^n \times \mathbf{M}_{\rm F}^n \times \mathbf{M}_{\rm S}^n \times \mathbf{Y}^n.$$

We can then rewrite the Eqs. (9) and (10) along with the continuity constraints (5) and (6) in the following fully-coupled three-field variational form. Find $(\overline{\mathbf{U}}^{n+1}, \overline{P}^{n+1}, \mathbf{D}^{n+1}, \mathbf{\Lambda}_{\rm F}^{n+1}, \mathbf{\Lambda}_{\rm S}^{n+1}, \mathbf{Z}^{n+1}) \in \mathscr{X}^n$ such that

$$a_{\rm F}^n(\overline{\mathbf{U}}^{n+1}, \mathbf{w}_{\rm F}) + b_{\rm F}^n(\overline{P}^{n+1}, \mathbf{w}_{\rm F}) + c_{\rm F}^n(\overline{\mathbf{U}}^{n+1}, \mathbf{w}_{\rm F}) + B_{\rm F}^n(\mathbf{\Lambda}_{\rm F}^{n+1}, \mathbf{w}_{\rm F}) = F_{\rm F}^n(\mathbf{w}_{\rm F}), \tag{11}$$

$$b_{\rm F}^n(w_{\rm D}, \overline{\mathbf{U}}^{n+1}) = 0, \tag{12}$$

$$a_{\rm S}(\mathbf{D}^{n+1}, \mathbf{w}_{\rm S}) + c_{\rm S}^n(\mathbf{D}^{n+1}, \mathbf{w}_{\rm S}) + B_{\rm S}(\mathbf{\Lambda}_{\rm S}^{n+1}, \mathbf{w}_{\rm S}) = F_{\rm S}^n(\mathbf{w}_{\rm S}), \tag{13}$$

$$\int_{\Gamma^n}(\overline{\mathbf{U}}^{n+1} - \mathbf{Z}^{n+1})\cdot\mathbf{\Psi}_1\,{\rm d}s = 0, \tag{14}$$

$$\int_{\Gamma^0}(\mathbf{D}^{n+1} - \Delta t\mathbf{Z}^{n+1})\cdot\mathbf{\Psi}_2\,{\rm d}s = \int_{\Gamma^0}\mathbf{D}^n\cdot\mathbf{\Psi}_2\,{\rm d}s \tag{15}$$

and

$$-\left(\int_{\Gamma^n} \mathbf{\Lambda}_{\mathrm{F}}^{n+1} \cdot \mathbf{\Phi}\,\mathrm{d}s + \int_{\Gamma^0} \mathbf{\Lambda}_{\mathrm{S}}^{n+1} \cdot \mathbf{\Phi}\,\mathrm{d}s\right) = 0 \tag{16}$$

$\forall(\mathbf{w}_{\mathrm{F}}, w_{\mathrm{D}}, \mathbf{w}_{\mathrm{S}}, \mathbf{\Psi}_1, \mathbf{\Psi}_2, \mathbf{\Phi}) \in \mathscr{X}^n$, where

$$a_{\mathrm{F}}^n(\mathbf{u}, \mathbf{w}_{\mathrm{F}}) = \nu \int_{\Omega_{\mathrm{F}}^n} \sum_{j=1}^{2} (\nabla u_j \cdot \nabla w_{\mathrm{F}}^{(j)})\,\mathrm{d}A,$$

$$b_{\mathrm{F}}^n(p, \mathbf{w}_{\mathrm{F}}) = -\int_{\Omega_{\mathrm{F}}^n} p(\nabla \cdot \mathbf{w}_{\mathrm{F}})\,\mathrm{d}A,$$

$$c_{\mathrm{F}}^n(\overline{\mathbf{U}}^{n+1}, \mathbf{w}_{\mathrm{F}}) = \int_{\Omega_{\mathrm{F}}^n} \sum_{j=1}^{2} \left[\frac{\overline{U}_j^{n+1}}{\Delta t} + (U_1^n - W^n)\partial_{x_1}\overline{U}_j^{n+1} + U_2^n \partial_{x_2}\overline{U}_j^{n+1}\right] w_{\mathrm{F}}^{(j)}\,\mathrm{d}A,$$

$$B_{\mathrm{F}}^n(\boldsymbol{\lambda}_{\mathrm{F}}, \mathbf{w}_{\mathrm{F}}) = -\int_{\Gamma^n} (\boldsymbol{\lambda}_{\mathrm{F}} \cdot \mathbf{w}_{\mathrm{F}})\,\mathrm{d}s,$$

$$F_{\mathrm{F}}^n(\mathbf{w}_{\mathrm{F}}) = \int_{\Omega_{\mathrm{F}}^n} \left(\overline{\mathbf{f}}^{n+1} + \frac{1}{\Delta t}\mathbf{U}^n\right) \cdot \mathbf{w}_{\mathrm{F}}\,\mathrm{d}A,$$

$$a_{\mathrm{S}}(\mathbf{d}, \mathbf{w}_{\mathrm{S}}) = \int_{\Omega_{\mathrm{S}}^0} \left[\mu \sum_{j=1}^{2} (\nabla d_j \cdot \nabla w_{\mathrm{S}}^j) + \epsilon(\nabla \cdot \mathbf{d})(\nabla \cdot \mathbf{w}_{\mathrm{S}})\right] \mathrm{d}A,$$

$$c_{\mathrm{S}}^n(\mathbf{d}^{n+1}, \mathbf{w}_{\mathrm{S}}) = \int_{\Omega_{\mathrm{S}}^0} \left(\frac{1}{(\Delta t)^2}\mathbf{d}^{n+1}\right) \cdot \mathbf{w}_{\mathrm{S}}\,\mathrm{d}A,$$

$$B_{\mathrm{S}}(\boldsymbol{\lambda}_{\mathrm{S}}, \mathbf{w}_{\mathrm{S}}) = -\int_{\Gamma^0} (\boldsymbol{\lambda}_{\mathrm{S}} \cdot \mathbf{w}_{\mathrm{S}})\,\mathrm{d}s,$$

$$F_{\mathrm{S}}^n(\mathbf{w}_{\mathrm{S}}) = \int_{\Omega_{\mathrm{S}}^0} \left(\mathbf{g}^{n+1} + \frac{1}{(\Delta t)^2}(2\mathbf{D}^n - \mathbf{D}^{n-1})\right) \cdot \mathbf{w}_{\mathrm{S}}\,\mathrm{d}A.$$

The next step in the finite element procedure is to define basis functions [23] for each of the finite dimensional spaces introduced and express the unknowns $(\overline{\mathbf{U}}^{n+1}, \overline{P}^{n+1}, \mathbf{D}^{n+1}, \mathbf{\Lambda}_{\mathrm{F}}^{n+1}, \mathbf{\Lambda}_{\mathrm{S}}^{n+1}, \mathbf{Z}^{n+1})$ as linear combinations of the respective basis functions. Choosing the test functions to be basis functions themselves then converts the above system of integral equations into a linear system that we solve for the appropriate coefficients in the linear combinations.

Hence, for each time step the three-field formulation developed allows us to simultaneously solve for the fluid velocity and structure displacement. The new interface position is then extrapolated and used to build the new mesh on the deformed fluid domain.

4. Consistency error

Let $\bar{\phi}^{n+1}(\mathbf{x}) = \phi^{n+1}(x_1 + w^n(\mathbf{x})\Delta t, x_2)$ for any function ϕ defined on $\Omega_{\mathrm{F}}(t^n)$. If necessary, we extend $\bar{\phi}^{n+1}$ analytically to Ω_{F}^n, shown in Fig. 3. Next, let

$$\bar{\mathbf{x}} = (x_1 + w^n(\mathbf{x})\Delta t, x_2) \quad \forall \mathbf{x} \in \Omega_{\mathrm{F}}^n.$$

Then $\bar{\phi}^{n+1}(\mathbf{x}) = \phi^{n+1}(\bar{\mathbf{x}})$.

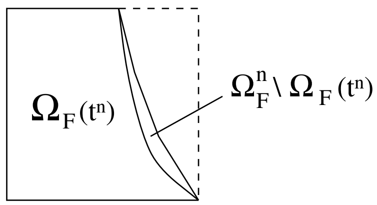

Fig. 3. Extension to Ω_F^n.

Theorem 1. *Let*

$$e_f^{n+1}(\mathbf{x}) = \sum_{j=1}^{2}\left(\frac{1}{\Delta t}(\bar{u}_j^{n+1} - u_j^n) - v\Delta\bar{u}_j^{n+1} + (u_1^n - w^n)\partial_{x_1}\bar{u}_j^{n+1} + u_2^n\partial_{x_2}\bar{u}_j^{n+1} + \partial_{x_j}\bar{p}^{n+1}\right)(\mathbf{x}) - \sum_{j=1}^{2}\bar{f}_j^{n+1}(\mathbf{x}).$$

If $\mathbf{u}$, p, *and* γ *are sufficiently smooth*, *then* $\exists C$ *such that*

$$\|e_f^{n+1}(\mathbf{x})\|_{L^\infty(\Omega_F^n)} \leqslant C\Delta t.$$

Before proving this theorem, a few lemmas are needed.

Lemma 2. *Let*

$$e_1^{n+1}(\mathbf{x}) = \sum_{j=1}^{2}\left[\frac{1}{\Delta t}(\bar{u}_j^{n+1} - u_j^n)(\mathbf{x}) - \partial_t u_j^{n+1}(\bar{\mathbf{x}}) - w^n(\mathbf{x})\partial_{x_1}u_j^{n+1}(\bar{\mathbf{x}})\right].$$

If $\mathbf{u}$ *is sufficiently smooth*, *then* $\exists\, C_1$ *such that*

$$\|e_1^{n+1}(\mathbf{x})\|_{L^\infty(\Omega_F^n)} \leqslant C_1\Delta t.$$

Proof. Using $u_j^{n+1}(\bar{\mathbf{x}}) = v_j^{n+1}(\mathbf{x}), u_j^n(\mathbf{x}) = v_j^n(\mathbf{x})$ and (7) we have,

$$\begin{aligned} e_1^{n+1}(\mathbf{x}) &= \sum_{j=1}^{2}\left[\frac{1}{\Delta t}(\bar{u}_j^{n+1} - u_j^n)(\mathbf{x}) - \partial_t v_j^{n+1}(\mathbf{x})\right] = \sum_{j=1}^{2}\left[\frac{1}{\Delta t}(v_j^{n+1} - v_j^n)(\mathbf{x}) - \partial_t v_j^{n+1}(\mathbf{x})\right] \\ &= \frac{\Delta t}{2}\sum_{j=1}^{2}\partial_{tt}v_j^{n+1}(\mathbf{x}) + \mathcal{O}(\Delta t^2). \end{aligned}$$

Employing the triangle inequality, the result is obtained. $\square$

Lemma 3. *Let*

$$e_2^{n+1}(\mathbf{x}) = v\sum_{j=1}^{2}\left[\Delta u_j^{n+1}(\bar{\mathbf{x}}) - \Delta\bar{u}_j^{n+1}(\mathbf{x})\right].$$

If $\mathbf{u}$ *and* γ *are sufficiently smooth*, *then* $\exists C_2$ *such that*

$$\|e_2^{n+1}(\mathbf{x})\|_{L^\infty(\Omega_F^n)} \leqslant C_2\Delta t.$$

Proof. Note that

$$\Delta\bar{u}_j^{n+1}(\mathbf{x}) = \Delta u_j^{n+1}(x_1 + w^n(\mathbf{x})\Delta t, x_2) = \nabla\cdot\left(\partial_{x_1}u_j^{n+1}(\bar{\mathbf{x}})\left(1+\frac{\dot\gamma(t,x_2)}{\gamma(t,x_2)}\Delta t\right), \partial_{x_2}u_j^{n+1}(\bar{\mathbf{x}})\right)$$
$$= \Delta u_j^{n+1}(\bar{\mathbf{x}}) + 2\partial_{x_1}^2 u_j^{n+1}(\bar{\mathbf{x}})\frac{\dot\gamma(t,x_2)}{\gamma(t,x_2)}\Delta t + \mathcal{O}(\Delta t^2),$$

so

$$e_2^{n+1}(\mathbf{x}) = -\left(2\nu\sum_{j=1}^{2}\partial_{x_1}^2 u_j^{n+1}(\bar{\mathbf{x}})\frac{\dot\gamma(t,x_2)}{\gamma(t,x_2)}\right)\Delta t + \mathcal{O}(\Delta t^2).$$

Under the assumption that the fluid–structure interface does not coincide with the left boundary of $\Omega_{\mathrm{F}}(t)$, the triangle inequality completes the proof. □

Lemma 4. *Let*

$$e_3^{n+1}(\mathbf{x}) = \sum_{j=1}^{2}\left(w^n(\mathbf{x})\partial_{x_1}u_j^{n+1}(\bar{\mathbf{x}}) + (u_1^n - w^n)(\mathbf{x})\partial_{x_1}\bar{u}_j^{n+1}(\mathbf{x}) + u_2^n(\mathbf{x})\partial_{x_2}\bar{u}_j^{n+1}(\mathbf{x}) - (\mathbf{u}^{n+1}\cdot\nabla u_j^{n+1})(\bar{\mathbf{x}})\right).$$

If $\mathbf{u}$ *and* γ *are sufficiently smooth, then* $\exists C_3$ *such that*

$$\|e_3^{n+1}(\mathbf{x})\|_{L^\infty(\Omega_{\mathrm{F}}^n)} \leqslant C_3\Delta t.$$

Proof. Note that

$$e_3^{n+1}(\mathbf{x}) = \sum_{j=1}^{2}\left(w^n(\mathbf{x})\left(\partial_{x_1}u_j^{n+1}(\bar{\mathbf{x}}) - \partial_{x_1}\bar{u}_j^{n+1}(\mathbf{x})\right) + (\mathbf{u}^n\cdot\nabla\bar{u}_j^{n+1})(\mathbf{x}) - (\mathbf{u}^{n+1}\cdot\nabla u_j^{n+1})(\bar{\mathbf{x}})\right).$$

It is easily seen that

$$\partial_{x_1}\bar{u}_j^{n+1}(\mathbf{x}) = \partial_{x_1}u_j^{n+1}(\bar{\mathbf{x}})\left(1+\frac{\dot\gamma(t,x_2)}{\gamma(t,x_2)}\Delta t\right)$$

and

$$(\mathbf{u}^n\cdot\nabla\bar{u}_j^{n+1})(\mathbf{x}) = [\mathbf{u}^n(\mathbf{x})\cdot\nabla u_j^{n+1}(\bar{\mathbf{x}})] + u_1^n(\mathbf{x})\partial_{x_1}u_j^{n+1}(\bar{\mathbf{x}})\frac{\dot\gamma(t,x_2)}{\gamma(t,x_2)}\Delta t.$$

Hence,

$$e_3^{n+1}(\mathbf{x}) = \sum_{j=1}^{2}\left(-w^n\frac{\dot\gamma(t,x_2)}{\gamma(t,x_2)}\Delta t + u_1^n(\mathbf{x})\partial_{x_1}u_j^{n+1}(\bar{\mathbf{x}})\frac{\dot\gamma(t,x_2)}{\gamma(t,x_2)}\Delta t\right) + \sum_{j=1}^{2}\left([\mathbf{u}^n(\mathbf{x}) - \mathbf{u}^{n+1}(\bar{\mathbf{x}})]\cdot\nabla u_j^{n+1}(\bar{\mathbf{x}})\right).$$

Now, noting that $\mathbf{u}^n(\mathbf{x}) - \mathbf{u}^{n+1}(\bar{\mathbf{x}}) = \mathcal{O}(\Delta t)$ and applying the L^∞ norm gives the desired result. □

Proof of Theorem 1. Note that

$$\bar{f}_j^{n+1}(\mathbf{x}) = f_j^{n+1}(\bar{\mathbf{x}}) = \partial_t u_j^{n+1}(\bar{\mathbf{x}}) - \nu\Delta u_j^{n+1}(\bar{\mathbf{x}}) + (\mathbf{u}^{n+1}\cdot\nabla u_j^{n+1})(\bar{\mathbf{x}}) + \partial_{x_j}p^{n+1}(\bar{\mathbf{x}}).$$

Hence,

$$e_f^{n+1}(\mathbf{x}) = e_1^{n+1}(\mathbf{x}) + e_2^{n+1}(\mathbf{x}) + e_3^{n+1}(\mathbf{x}) + \sum_{j=1}^{2}[\partial_{x_j}\bar{p}^{n+1}(\mathbf{x}) - \partial_{x_j}p^{n+1}(\bar{\mathbf{x}})].$$

Using Lemmas 2–4 and observing that

$$\partial_{x_j}\bar{p}^{n+1}(\mathbf{x}) - \partial_{x_j}p^{n+1}(\bar{\mathbf{x}}) = \mathcal{O}(\Delta t),$$

the proof is easily completed. □

For the structure, a similar theorem can be proved for the consistency error associated with the numerical scheme in (10). We state in the following theorem the main result.

Theorem 5. *Let*

$$e_s^{n+1}(\mathbf{x}) = \sum_{j=1}^{2}\left[\frac{1}{\Delta t^2}(d_j^{n+1} - 2d_j^n + d_j^{n-1}) - \mu\Delta d_j^{n+1} - \varepsilon\partial_{x_j}(\nabla\cdot\mathbf{d}^{n+1})\right](\mathbf{x}) - \sum_{j=1}^{2} g_j^{n+1}(\mathbf{x}).$$

If $\mathbf{d}$ *is sufficiently smooth, then* $\exists C$ *such that*

$$\|e_s^{n+1}(\mathbf{x})\|_{L^\infty(\Omega_S^0)} \leqslant C\Delta t.$$

5. Stability

From Eqs. (11)–(13),

$$\begin{aligned} &a_F^n(\overline{\mathbf{U}}^{n+1}, \mathbf{w}_F) + b_F^n(\overline{P}^{n+1}, \mathbf{w}_F) + c_F^n(\overline{\mathbf{U}}^{n+1}, \mathbf{w}_F) + B_F^n(\mathbf{\Lambda}_F^{n+1}, \mathbf{w}_F) + b_F^n(w_D, \overline{\mathbf{U}}^{n+1}) + a_S(\mathbf{D}^{n+1}, \mathbf{w}_S) \\ &\quad + c_S^n(\mathbf{D}^{n+1}, \mathbf{w}_S) + B_S(\mathbf{\Lambda}_S^{n+1}, \mathbf{w}_S) = F_F^n(\mathbf{w}_F) + F_S^n(\mathbf{w}_S) \end{aligned} \tag{17}$$

$\forall(\mathbf{w}_F, w_D, \mathbf{w}_S) \in \mathbf{V}_F^n \times W_F^n \times \mathbf{V}_S^n$. We define

$$\|\mathbf{U}\|_{L^2(\Omega_F^n)} = \left(\int_{\Omega_F^n} (\mathbf{U}\cdot\mathbf{U})^2\,\mathrm{d}A\right)^{\frac{1}{2}}.$$

Let $\mathbf{w}_F = \Delta t\overline{\mathbf{U}}^{n+1}$, $w_D = \Delta t\overline{P}^{n+1}$, and $\mathbf{w}_S = \mathbf{D}^{n+1} - \mathbf{D}^n$. Using these choices for our test functions, we arrive at the following energy estimate.

Theorem 6. *Let* $\boldsymbol{\delta}^{n+1} = \mathbf{D}^{n+1} - \mathbf{D}^n$. *Then* $\exists C > 0$, *independent of* Δt, *such that*

$$\begin{aligned} &\nu\Delta t\sum_{k=0}^{n}\sum_{j=1}^{2}\|\nabla\overline{U}_j^{k+1}\|^2_{L^2(\Omega_F^k)} + \|\mathbf{U}^{n+1}\|^2_{L^2(\Omega_F^{n+1})} + \mu\sum_{j=1}^{2}\|\nabla D_j^{n+1}\|^2_{L^2(\Omega_S^0)} + \varepsilon\int_{\Omega_S^0}|\nabla\cdot\mathbf{D}^{n+1}|^2\,\mathrm{d}A + \frac{1}{(\Delta t)^2}\|\boldsymbol{\delta}^{n+1}\|^2_{L^2(\Omega_S^0)} \\ &\quad\leqslant C\Bigg[\|\mathbf{U}^0\|^2_{L^2(\Omega_F^0)} + \mu\sum_{j=1}^{2}\|\nabla D_j^0\|^2_{L^2(\Omega_S^0)} + \varepsilon\int_{\Omega_S^0}|\nabla\cdot\mathbf{D}^0|^2\,\mathrm{d}A + \frac{1}{(\Delta t)^2}\|\boldsymbol{\delta}^0\|^2_{L^2(\Omega_S^0)} \\ &\qquad + \sum_{k=0}^{n}\left(\Delta t\|\overline{\mathbf{f}}^{k+1}\|^2_{L^2(\Omega_F^k)} + \|\mathbf{g}^{k+1}\|^2_{L^2(\Omega_S^0)}\right)\Bigg]. \end{aligned}$$

As in the previous section, we will utilize a few lemmas in the proof of this result.

Lemma 7

$$b_{\mathrm{F}}^{n}(\overline{P}^{n+1},\Delta t\overline{\mathbf{U}}^{n+1})+B_{\mathrm{F}}^{n}(\mathbf{\Lambda}_{\mathrm{F}}^{n+1},\Delta t\overline{\mathbf{U}}^{n+1})+b_{\mathrm{F}}^{n}(\Delta t\overline{P}^{n+1},\overline{\mathbf{U}}^{n+1})+B_{\mathrm{S}}(\mathbf{\Lambda}_{\mathrm{S}}^{n+1},\boldsymbol{\delta}^{n+1})=0.$$

Proof. Note that

$$\begin{aligned}&2\Delta t b_{\mathrm{F}}^{n}(\overline{P}^{n+1},\overline{\mathbf{U}}^{n+1})+\Delta t B_{\mathrm{F}}^{n}(\mathbf{\Lambda}_{\mathrm{F}}^{n+1},\overline{\mathbf{U}}^{n+1})+B_{\mathrm{S}}(\mathbf{\Lambda}_{\mathrm{S}}^{n+1},\boldsymbol{\delta}^{n+1})\\&\quad=2\Delta t b_{\mathrm{F}}^{n}(\overline{P}^{n+1},\overline{\mathbf{U}}^{n+1})-\int_{\Gamma^{n}}\mathbf{\Lambda}_{\mathrm{F}}^{n+1}\cdot(\Delta t\overline{\mathbf{U}}^{n+1})\,\mathrm{d}s-\int_{\Gamma^{0}}\mathbf{\Lambda}_{\mathrm{S}}^{n+1}\cdot\boldsymbol{\delta}^{n+1}\,\mathrm{d}s.\end{aligned}$$

But $\overline{P}^{n+1}\in W_{\mathrm{F}}^{n}$, so $b_{\mathrm{F}}^{n}(\overline{P}^{n+1},\overline{\mathbf{U}}^{n+1})=0$. And letting $\mathbf{\Psi}_1=\mathbf{\Lambda}_{\mathrm{F}}^{n+1}$, $\mathbf{\Psi}_2=\Lambda_{\mathrm{S}}^{n+1}$, and $\mathbf{\Phi}=\mathbf{Z}^{n+1}$ in Eqs. (14)–(16), we have that

$$-\int_{\Gamma^{n}}(\Delta t\mathbf{Z}^{n+1})\cdot\mathbf{\Lambda}_{\mathrm{F}}^{n+1}\,\mathrm{d}s-\int_{\Gamma^{0}}(\Delta t\mathbf{Z}^{n+1})\cdot\mathbf{\Lambda}_{\mathrm{S}}^{n+1}\,\mathrm{d}s=-\Delta t\left(\int_{\Gamma^{n}}\mathbf{\Lambda}_{\mathrm{F}}^{n+1}\cdot\mathbf{\Phi}\,\mathrm{d}s+\int_{\Gamma^{0}}\mathbf{\Lambda}_{\mathrm{S}}^{n+1}\cdot\mathbf{\Phi}\,\mathrm{d}s\right)=0.\qquad\square$$

Lemma 8

$$\Delta t c_{\mathrm{F}}^{n}(\overline{\mathbf{U}}^{n+1},\overline{\mathbf{U}}^{n+1})=\frac{1}{2}\|\overline{\mathbf{U}}^{n+1}\|^{2}_{L^{2}(\Omega_{\mathrm{F}}^{n})}+\frac{1}{2}\|\mathbf{U}^{n+1}\|^{2}_{L^{2}(\Omega_{\mathrm{F}}^{n+1})}.$$

Proof

$$\Delta t c_{\mathrm{F}}^{n}(\overline{\mathbf{U}}^{n+1},\overline{\mathbf{U}}^{n+1})=\|\overline{\mathbf{U}}^{n+1}\|^{2}_{L^{2}(\Omega_{\mathrm{F}}^{n})}+\Delta t\sum_{j=1}^{2}\int_{\Omega_{\mathrm{F}}^{n}}[(U_1^n-W^n)\partial_{x_1}\overline{U}_j^{n+1}+U_2^n\partial_{x_2}\overline{U}_j^{n+1}]\overline{U}_j^{n+1}\,\mathrm{d}A,$$

but using the fact that $\mathbf{U}^n=\mathbf{W}^n$ on the left and right boundaries of Ω_{F}^{n}, where $\mathbf{W}^n=(W^n,0)$, this last term can be written as

$$\begin{aligned}&\frac{\Delta t}{2}\sum_{j=1}^{2}\left[\int_{\Omega_{\mathrm{F}}^{n}}\mathbf{U}^n\cdot\nabla(\overline{U}_j^{n+1})^2\,\mathrm{d}A-\int_{\Omega_{\mathrm{F}}^{n}}W^n\partial_{x_1}(\overline{U}_j^{n+1})^2\,\mathrm{d}A\right]\\&\quad=-\frac{\Delta t}{2}\left[\int_{\Omega_{\mathrm{F}}^{n}}(\nabla\cdot\mathbf{U}^n)[(\overline{U}_1^{n+1})^2+(\overline{U}_2^{n+1})^2]\,\mathrm{d}A-\sum_{j=1}^{2}\int_{\Omega_{\mathrm{F}}^{n}}\partial_{x_1}W^n(\overline{U}_j^{n+1})^2\,\mathrm{d}A\right]\end{aligned}$$

and $[(\overline{U}_1^{n+1})^2+(\overline{U}_2^{n+1})^2]\in W_{\mathrm{F}}^{n}$, so

$$\begin{aligned}\Delta t c_{\mathrm{F}}^{n}(\overline{\mathbf{U}}^{n+1},\overline{\mathbf{U}}^{n+1})&=\|\overline{\mathbf{U}}^{n+1}\|^{2}_{L^{2}(\Omega_{\mathrm{F}}^{n})}+\frac{\Delta t}{2}\sum_{j=1}^{2}\int_{\Omega_{\mathrm{F}}^{n}}\partial_{x_1}W^n(\overline{U}_j^{n+1})^2\,\mathrm{d}A\\&=\frac{1}{2}\|\overline{\mathbf{U}}^{n+1}\|^{2}_{L^{2}(\Omega_{\mathrm{F}}^{n})}+\frac{1}{2}\sum_{j=1}^{2}\int_{\Omega_{\mathrm{F}}^{n}}(1+\Delta t\partial_{x_1}W^n)(\overline{U}_j^{n+1})^2\,\mathrm{d}A\\&=\frac{1}{2}\|\overline{\mathbf{U}}^{n+1}\|^{2}_{L^{2}(\Omega_{\mathrm{F}}^{n})}+\frac{1}{2}\|\mathbf{U}^{n+1}\|^{2}_{L^{2}(\Omega_{\mathrm{F}}^{n+1})}.\qquad\square\end{aligned}$$

Proof of Theorem 6. Using our chosen test functions and applying Lemma 7, Eq. (17) becomes

$$\begin{aligned}&\Delta t a_{\mathrm{F}}^{n}(\overline{\mathbf{U}}^{n+1},\overline{\mathbf{U}}^{n+1})+\Delta t c_{\mathrm{F}}^{n}(\overline{\mathbf{U}}^{n+1},\overline{\mathbf{U}}^{n+1})+a_{\mathrm{S}}(\mathbf{D}^{n+1},\boldsymbol{\delta}^{n+1})+c_{\mathrm{S}}^{n}(\mathbf{D}^{n+1},\boldsymbol{\delta}^{n+1})\\&\quad=\Delta t\int_{\Omega_{\mathrm{F}}^{n}}\overline{\mathbf{f}}^{n+1}\cdot\overline{\mathbf{U}}^{n+1}\,\mathrm{d}A+\int_{\Omega_{\mathrm{F}}^{n}}\mathbf{U}^n\cdot\overline{\mathbf{U}}^{n+1}\,\mathrm{d}A+\int_{\Omega_{\mathrm{S}}^{0}}\mathbf{g}^{n+1}\cdot\boldsymbol{\delta}^{n+1}\,\mathrm{d}A+\frac{1}{(\Delta t)^2}\int_{\Omega_{\mathrm{S}}^{0}}\boldsymbol{\delta}^{n+1}\cdot\boldsymbol{\delta}^{n}\,\mathrm{d}A\\&\qquad+\frac{1}{(\Delta t)^2}\int_{\Omega_{\mathrm{S}}^{0}}\mathbf{D}^n\cdot\boldsymbol{\delta}^{n+1}\,\mathrm{d}A.\end{aligned}$$

Applying Young's inequality,

$$\Delta t \int_{\Omega_{\mathrm{F}}^{n}} \overline{\mathbf{f}}^{n+1} \cdot \overline{\mathbf{U}}^{n+1}\, \mathrm{d}A \leqslant C\Delta t \|\overline{\mathbf{f}}^{n+1}\|_{L^2(\Omega_{\mathrm{F}}^n)} \sum_{j=1}^{2} \|\nabla \overline{U}_j^{n+1}\|_{L^2(\Omega_{\mathrm{F}}^n)} \leqslant C\Delta t \|\overline{\mathbf{f}}^{n+1}\|^2_{L^2(\Omega_{\mathrm{F}}^n)} + \frac{\nu \Delta t}{2} \sum_{j=1}^{2} \|\nabla \overline{U}_j^{n+1}\|^2_{L^2(\Omega_{\mathrm{F}}^n)},$$

where C is independent of Δt, application of Lemma 8 and the Schwarz inequality gives

$$\begin{aligned}
&\nu \Delta t \sum_{j=1}^{2} \|\nabla \overline{U}_j^{n+1}\|^2_{L^2(\Omega_{\mathrm{F}}^n)} + \|\mathbf{U}^{n+1}\|^2_{L^2(\Omega_{\mathrm{F}}^{n+1})} + \mu \sum_{j=1}^{2} \|\nabla D_j^{n+1}\|^2_{L^2(\Omega_{\mathrm{S}}^0)} + \varepsilon \int_{\Omega_{\mathrm{S}}^0} |\nabla \cdot \mathbf{D}^{n+1}|^2\, \mathrm{d}A + \frac{1}{(\Delta t)^2} \|\boldsymbol{\delta}^{n+1}\|^2_{L^2(\Omega_{\mathrm{S}}^0)} \\
&\quad \leqslant \|\mathbf{U}^n\|^2_{L^2(\Omega_{\mathrm{F}}^n)} + \mu \sum_{j=1}^{2} \|\nabla D_j^n\|^2_{L^2(\Omega_{\mathrm{S}}^0)} + \varepsilon \int_{\Omega_{\mathrm{S}}^0} |\nabla \cdot \mathbf{D}^n|^2\, \mathrm{d}A + \frac{1}{(\Delta t)^2} \|\boldsymbol{\delta}^n\|^2_{L^2(\Omega_{\mathrm{S}}^0)} \\
&\qquad + C\Delta t \|\overline{\mathbf{f}}^{n+1}\|^2_{L^2(\Omega_{\mathrm{F}}^n)} + \|\mathbf{g}^{n+1}\|^2_{L^2(\Omega_{\mathrm{S}}^0)}.
\end{aligned}$$

Applying the discrete Gronwall inequality [12], the desired result is obtained. □

6. Conclusion

Given bounded initial and boundary conditions, the method proposed has been shown to be both consistent and stable. We also expect to demonstrate exponential convergence for the technique in the presence of nonquasiuniform meshes. The latter aspect will be considered in a following paper which will include numerical results which confirm the theory presented herein.

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