

Nonlinear models for biologically-inspired elastic membrane wings

Edward W. Swim*

United States Military Academy, West Point, NY, 10996

Padmanabhan Seshaiyer†

George Mason University, Fairfax, VA, 22030

Nonlinear membrane wing models that mimic the composition and flexibility of biological tissues such as those found in the wings of bats and insects can be valuable in the development of structural components for micro air vehicles. In this work, we develop a framework for building fluid-structure interaction simulations that allow for both geometric and material nonlinearity within a membrane model. A nonconforming finite element method is used to allow for independent mesh construction and polynomial degree of approximation within each subdomain. Moreover, an arbitrary Lagrangian-Eulerian formulation is used to manipulate the meshes at each time step. Both steady-state and dynamic models are tested against data from the literature.

Nomenclature

\mathbf{C}	right Cauchy-Green tensor
E	Young's modulus of elasticity
I_j	strain invariants
L	membrane length
L_0	initial membrane length
\mathbf{n}	unit normal vector
\mathbf{S}	nominal strain tensor
T	surface tension
u	axial membrane displacement
v_j	fluid velocity component
z	transverse membrane displacement
ε	strain of the middle line
κ	curvature of the middle line
μ	dynamic fluid viscosity
ρ_f	fluid density
ρ_m	membrane density
$\boldsymbol{\sigma}$	Cauchy stress tensor
τ	fluid shear stress at fluid-membrane interface

I. Introduction

Micro air vehicles (MAVs), which typically have a wing span of at most six inches, flight speeds of twenty to forty miles per hour, and a total weight less than half a pound,^{8,13} encounter flight conditions with Reynolds numbers similar to that of an insect or bird ($10^3 - 10^5$). As a result, stall may occur for moderate

*Assistant Professor, Department of Mathematical Sciences, Member AIAA.

†Associate Professor, Department of Mathematical Sciences, Member AIAA.