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Title: The international journal for technology in mathematics education.

Uniform Title:

Author:

Edition:

Imprint: Hemel Hempstead, Eng. : Research Information Ltd., 2004-

Article: : Student Misconceptions Caused by Misuse of Technology

Vol: 14 No.: 4 Pages: 189-196 Date: 2007

Dissertation:

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Student Misconceptions Caused by Misuse of Technology

By Robert Paige, Padmanabhan Seshaiyer and Magdalena Toda

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Received: 10 January 2006 Revised: October 18, 2006

Calculators used widely by students, teachers, scientists, engineers and many others provide an interesting case study of a compelling technology that has helped change the way many professionals work. They not only help in enhancing problem solving skills of most individuals, but also help visualise solutions to problems in a better way. Research supports the claim that calculator use improves student performance in computation, concept development, and problem-solving although a growing number of studies show that there may be a class of errors and misconceptions that are induced by calculators. We review some basic ideas of errors in numerical analysis and discuss in detail the concept of round-off error that is often noticed by both college teachers and high school and undergraduate students when working with such computing aids. We then present experimental results on the performance of a variety of computing aids for solving two problems, perform a statistical analysis of data collected from 215 students in the freshmen calculus class at Texas Tech University, and report the findings of this analysis.

1 INTRODUCTION

Calculators used widely by students, teachers, scientists, engineers and many more provide an interesting case study of a compelling technology that has helped change the way many professionals work. They not only help in enhancing problem solving skills of most individuals, but also help visualise solutions to problems in a better way. The National Council of Teachers of Mathematics (NCTM) has advocated that the calculators be used by students to explore and experiment mathematical ideas; develop and reinforce computational skills and analysing data; assist in problem-solving; perform complex computations of real-world problems; gain access to mathematical concepts that go beyond those limited by traditional paper-and-pencil computation (NCTM, 1991). Further, the same report recommends that every teacher promotes the usage of calculators in mathematics instruction. The advantages of using calculators in improving basic arithmetic and problem solving skills from an early age have been well documented (Suydam, 1982; Hembree and Dessart, 1986). Research also indicates that students using calculators show a better attitude toward mathematics and also show greater ease in problem solving (Hembree and Dessart, 1986; Heid, 1997; Dunham and Dick, 1994; Smith, 1997). Currently, calculators are being used in most classrooms and there has been a lot of interest in understanding how calculators can be used efficiently by students (Futch and Stephens 1997; Porter 1991; Spath 1990; Tan, 1995; Berry, Graham and Smith, 2005). Although research supports the claim that calculator use improves student performance in computation, concept development, and problem-solving, a growing number of studies show that there may be a class of errors and misconceptions that are induced by calculators (Tuska, 1993; Goldenberg, 1988; Steele, 1995; Dunham and Osborne, 1991; Berry, Graham and Smith, 2003).

Through our research in numerical computing, we have come to appreciate that analysing a numerical technique requires a combined theoretical and computational approach. Theory is needed to guide the performance and interpretation of the numerical techniques, while computation is necessary to verify the robustness and stability of the numerical technique. Not implementing any numerical technique in a correct and sensible fashion can lead to major disasters. One such example is the Patriot Missile failure, in Dharan, Saudi Arabia, on February 25, 1991 (GAO/IMTEC-92-96, 1992). This incident resulted in 28 deaths and was ultimately attributable to poor handling of inexact computer round-off arithmetic. Another example is the explosion of the Ariane 5 rocket just 40 seconds after lift-off on its maiden voyage off French Guiana, on June 4, 1996 (Ariane 501, 1996; Gleick, 1996). The destroyed rocket and its cargo were valued at $500 million. It has been our experience that these kinds of problems also arise in a day-to-day mathematics classroom with just a desk calculator or a digital computer, although not on such a large scale as the Patriot Missile or Ariane 5 failures.

The objective of this paper is to present research evidence supporting the need for teaching students the importance of using calculators effectively. We will first review the concept of round-off error that is often noticed by both college teachers and high school and undergraduate students when working with such computing aids. We then present experimental results on the performance of a variety of computing aids for solving two problems. Finally, we perform a statistical analysis of data collected from 215 students in the freshmen calculus class at Texas Tech University and report the findings of this analysis.

We sincerely hope that this article will help students and teachers understand these errors better and also become more aware of the limitations of various computing aids that are used in a traditional classroom setting.

2 RESEARCH METHODOLOGY

Our motivation to write this article came when a few students in our Calculus class approached us with a similar problem. The students were asked to evaluate the following limit, using their respective calculators:

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x \qquad (1)$$

Although some students obtained the "correct" answer, others (whose calculators did not accommodate ∞) approximated ∞ by large values which was a reasonable thing to do. To their surprise, they found that above a certain specific approximation to ∞ (which differed for each student using a different calculator), their answers were no longer approaching the true value *e*. Instead, they obtained the incorrect answer 1. Naturally, as any calculus student would, they got confused and came to various conclusions, some of which were; (a) they must be doing something wrong, (b) the teacher must have made a mistake, (c) their calculator was no good and they needed to buy a better one.

When this problem was presented to us by the students, we pointed out, of course, that the problem was because of inexact calculator arithmetic. We also realised that for a student, the first time such a phenomenon is introduced is usually in a senior-level numerical analysis course. This concept is well-known, (see Burden and Faires, 2000, Kincaid and Cheney, 2002) and it has been a prominent research area in the field of numerical analysis. On the other hand, most calculator users (including a majority of undergraduate and high school students) are not aware of this. In fact, most students at these levels believe that computers and calculators cannot make mistakes by themselves, and therefore attribute these to human errors. To avoid this, we believe that it is not enough to just teach these students how to use any such computing aid, but also teach them about the limitations of such aids in numerical computing.

3 ACCURACY OF NUMERICAL COMPUTATIONS

To understand the flaw in the previous section and most other everyday numerical quirks, one needs to know a few basic facts about how a machine stores numbers and does the arithmetic. There are two kinds of answers, *exact* and *approximate*. We can approximate any real number to a certain degree of accuracy. Approximate numbers are those that represent the numbers to a certain degree of accuracy.

For example, an approximate value of *e* is 2.718, or if we desire a better approximation, e.g., 2.71828182845. The digits that are used to express a number on a machine are called *significant digits* and thus, the number 2.718 contains four significant digits. The number 0.0027 has, however, only two significant digits, namely 2 and 7, since the zeros serve only to fix the position of the decimal point.

Different machines may allocate a different number of significant digits, but the essential feature is that the number of significant digits is finite. Therefore, even when the initial data are all exact numbers, the results of calculations need not be.

Consider a *k*-digit machine which uses *k* significant digits to represent any given exact number. There are two standard procedures which may be employed to generate a *k*-digit representation of a number. The first is *chopping* wherein the machine records only the first *k* significant digits of a given number. For example, on a 3-digit machine 2.714 and 2.716 are represented as 2.71. The second procedure is *rounding*. To round-off a number to *N* significant digits, we discard all digits to the right of the *N* th digit, and if the (*N* + 1) th digit is greater than or equal to 5, we add 1 to the *N* th digit; otherwise we merely retain the first *N* significant digits. For example 2.714 and 2.716 are rounded-off to the three significant figures 2.71 and 2.72, respectively.

Suppose that x is the exact value of a number and x^* is its machine representation which is often referred to as the floating point approximation. Note that there are many different floating point number systems (Matsui and Iri, 1981; Matula, 1969), each of which uses a finite number of digits to represent x^*. Let the absolute error be defined as $|x-x^*|=\Delta x$.

Consider the expression (1). Let us now estimate the error committed in evaluating the function

$$u(x)=\left(1+\frac{1}{x}\right)^x.$$

The approximate value of the function u^* (where Δu is the absolute error in u) is given by $u^*=u(x+\Delta x)$. Expanding the right hand side $u(x+\Delta x)$ by Taylor's series, we obtain,

$$u^*=u(x)+\frac{du}{dx}\Delta x+\textit{terms involving }(\Delta x)^2$$

Assuming that the errors in x are small and that $\Delta x \ll 1$, so that the squares and higher powers of Δx can be neglected, the above relation yields,

$$u^*\approx u+\left(\frac{-1/x}{1+1/x}+\ln(1+1/x)\right)(1+1/x)^x\Delta x. \qquad (2)$$

Let $\Delta u=|u^*-u|$ and $e(x)=\left(\frac{-1/x}{1+1/x}+\ln(1+1/x)\right)x.$

Rearranging (2) and taking absolute values yields,

$$|\Delta u| \approx \frac{|e(x)|}{|x|}\left|\left(1+\frac{1}{x}\right)^x\right||\Delta x|.$$

Noting that $u(x)=(1+1/x)^x$ we have,

$$\left|\frac{\Delta u}{u}\right| = |e(x)|\left|\frac{\Delta x}{x}\right|. \qquad (3)$$

From (3), it is clear that the relative error in the function evaluation of $u(x)$ given by $\left|\frac{\Delta u}{u}\right|$ depends on how accurately x is represented on the machine. As $x\to\infty$ we have,

$$\begin{aligned}\lim_{x\to\infty} e(x) &= \lim_{x\to\infty}\left(\frac{-1/x}{1+1/x}+\ln(1+1/x)\right)x \\ &= \lim_{x\to\infty}\left(\frac{-x}{1+x}+\ln(1+1/x)^x\right) \\ &= \lim_{x\to\infty}\frac{-x}{1+x}+\ln\left(\lim_{x\to\infty}(1+1/x)^x\right) \\ &= -1+\ln e = 0\end{aligned}$$

Hence, ideally as $x\to\infty$ (3) yields Δu=0 i.e. $u^* \approx u$. However, on a calculator that accurately represents N decimal digits, $x=10^{N+K}$ $(k \ge 1)$ would be represented as 10^N, and ultimately $1+\frac{1}{x}$ is represented as the number 1. This of course is incorrect and is due to the addition of a tiny number to a large number. The smallest number that can be added to 1 such that the result is greater than 1 using finite precision is referred to as the *machine epsilon* and is often denoted as *eps*. MATLAB uses 16 digits and therefore one would expect *eps* to be close to 10^{-16}. Hence one can expect the MATLAB computations to become incorrect when $N \approx 16$. This is illustrated in our numerical results.

4 NUMERICAL EXPERIMENTS

In order to test the theory that was presented in the last section, we gave the following exercise to our students. We asked them to evaluate, $\left(1+10^{-2N}\right)^{10^{2N}}$ for N = 1,...,8,10 using several computing aids. The results are illustrated in Table 1. It is clear from this table that after N = 7 most frequently used calculators start yielding incorrect results. Table 1 clearly illustrates the importance of an awareness of the limitations of each machine used.

Let us now consider a second example of calculating

$$I=\int_0^\infty e^{-x}\,dx$$

using a TI-85 calculator. Let us assume that we approximate ∞ by N, where N is a large number. The value of the integral for various increasing values of N is then given in Table 2.

N	200	400	600	800	1000	1200
I	1	1	1	0	0	0

Table 2 Integral Values Using a TI-85 Calculator

The reason for these erroneous results is obviously dependent on how the TI-85 performs the integral computation. For example, most machines compute integrals numerically via some specific quadrature rule. It is, however, important to understand that every numerical scheme comes with an error and this error depends on the type of approximation that is assumed. From the table above, one can see that for increasing values of N, which is an approximation to ∞, one obtains incorrect results. To do a more detailed numerical computation, try N = 769 and N = 770 which yield different results. This example clearly indicates the effect of round-off and it also suggests that the value of N need not be very high for computations to become incorrect.

N	MATLAB	Casio Fx-7400G	TI-92	TI-85	TI-83	TI-30x	Casio Fx-115s	TI-25x
1	2.70481	2.70481	2.70481	2.70481	2.70481	2.70481	2.70481	2.70481
2	2.71815	2.71815	2.71815	2.71815	2.71815	2.71815	2.71815	2.71815
3	2.71828	2.71828	2.71828	2.71828	2.71828	2.71828	2.71828	2.71828
4	2.71828	2.71828	2.71828	2.71828	2.71828	2.71837	2.71828	1
5	2.71828	2.71828	2.71828	2.71828	2.71828	2.72772	2.71828	1
6	2.71852	2.71828	2.71828	2.71828	2.71828	1	1	1
7	2.71611	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1

Table 1 *Evaluation of* $\left(1+10^{-2N}\right)^{10^{2N}}$ *for* N=1,...,8,10 *on various hand-held calculators.*

Our claim is further substantiated by the following simple example of solution to linear systems. Consider solving the system, $A\,x = b$ where,

$$A=\begin{bmatrix} 0.00000001 & 10000000.0 \\ 0.0000000201 & 20000000.0 \end{bmatrix} \quad \text{and}$$

$$b=[10000000.0 \quad 20000000.01]^T.$$

Although, the exact solution is $x=[0 \quad 1]^T$, most commonly used technological tools perform incorrect computations when solving this system, due to the respective machine limits. For example, if the solution is obtained as $x = A^{-1}b$ from MATLAB one obtains $x=[-32 \quad 1]^T$. This example can be used to demonstrate the importance of *condition number* and invertibility of a matrix. Incidentally, the condition number $k(A)$ of matrix A can be defined as $k(A) := \|A\| \|A^{-1}\|$.

5 STATISTICAL ANALYSIS

In our study, a calculator exercise (see the Appendix) was administered to 215 students in the freshmen calculus class at Texas Tech University. From the completed sheets the following information was extracted for a statistical analysis; (i) sex of the student, (ii) name and brand of calculator used in the experiment, and (iii) whether or not the student was fooled by their calculator's erroneous output into thinking limit L is something other than e. Here a variable called "fooled" was used. This variable could assume values "yes" or "no", or was simply left blank if the student did not indicate whether or not they were fooled by erroneous output from their calculator. Variable fooled was also left blank in the few rare instances a calculator yielded the correct value for limit L. In the data set, results were reported for 19 different types of calculators. Many of these different types of calculators were used by only one or two students in the study and the overwhelming majority of students used a TI-83, TI-86 or TI-89 model calculator. Therefore, in our analysis we used four categories of calculators: TI-83, TI-86, TI-89 and "other". Here, a student's calculator is classified as belonging to the "other" category if it was is not a TI-83, TI-86 or TI-89 model.

In our analysis, which was performed in SAS version 9.1.4, we first considered an extended version of Fisher's exact test known as the Freeman-Halton test for nonrandom associations between variables fooled and calculator type. This test yielded a p-value of 0.0572 which is not statistically significant in the strict sense but is suggestive of an association between variables fooled and calculator. Next, we controlled for sex in Fisher's exact test for variables fooled and calculator type. This means that one Fisher's exact test was performed on data from the female students and another Fisher's exact test was performed on data from the male students. The test for the female students yielded a p-value of 1.0 suggesting that there is no non-random association between variables fooled and calculator type for female students. On the other hand the test for the male students yielded a p-value of 0.0070. This suggests that there is some form of non-random association between variables fooled and calculator type for the male students. To understand the nature of this association, a table of expected and observed percentages for males was generated in SAS. The expected percentages were computed assuming that there is no non-random association between variables fooled and calculator type. These computations are summarised in Table 3 (expected percentages are shown in parentheses). From this table we see the greatest disparity between expected and observed percentages for the TI-86 and TI-89 calculators, which are the most modern calculators of all those appearing in the study. This result may also suggest that male students are more easily misled by technology. Interestingly enough male students were less likely to be misled by the more sophisticated TI-89 than by the relatively less sophisticated TI-86. As pointed out by a referee, this phenomenon may be an artefact of the fact that the TI-89 is a handheld computer algebra system whereas the TI-86 is a graphing calculator.

	Fooled	Not Fooled
TI-83	19.44% (17.69%)	16.67% (21.31%)
TI-86	6.48% (3.63%)	0.93% (4.37%)
TI-89	6.48% (12.70%)	19.44% (15.30%)
Other	3.70% (4.99%)	6.48% (6.01%)

Freeman-Halton p-value = 0.0070

Table 3 *Table of observed and expected percentages for males. Expected percentages are shown in parentheses.*

6 CONCLUSIONS

Although the simple examples and associated statistical analyses illustrated in this article give an understanding of the limitations of machine precision, there are ways to improve this precision. Several prominent mathematical packages such as Mathematica, MATLAB, and Maple etc. have incorporated ways to vary the precision of the machine by allowing the user to set the number of significant digits for exact representation. The ability to reduce the effects of round-off error by raising the precision of a calculation is certainly very useful, but it is far from a universal solution to all problems with numerical error. High precision calculations normally use more time and memory than low precision calculations.

Over the last several years, there have been dramatic improvements in computing power, evolution of languages, tremendous growth in the size of data sets and an increase in the complexity of application requirements. The introduction of integrated development environments such as sophisticated calculators/computers and other software tools has improved productivity, especially in teaching college mathematics. However, the development of a sophisticated, robust analytical application still requires a major investment of time and money.

Therefore, the message that we would like to broadcast is that it is possible for any pure numerical algorithm, including the ones used on most desk calculators or digital computers, to produce incorrect results, and to do so without warning. The only way to be certain that the numerical results are correct is to be more cautious and perhaps adopt a sceptical attitude towards them until they can be verified by independent analysis. This may include detailed investigation of the algorithm that was used to do the calculation, and an understanding the limitations of the machine on which the calculations are performed.

ACKNOWLEDGEMENTS

We would like to thank the two anonymous referees whose suggestions and comments improved the form and content of our paper.

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BIOGRAPHICAL NOTES

Dr. Robert L. Paige is an Assistant Professor at Texas Tech University. His research interests include the interface between artificial neural networks and mainstream statistics, saddlepoint approximations, nonlinear regression models, statistical learning theory, survey sampling and nonparametric statistics. Some of his past research has addressed Bayesian inference in neural networks and lumpability of the stochastic Hopfield model. His present research concerns wavelet neural networks and small sample inference via saddlepoint approximations. He was a principal organiser of the 2005 Redraider Minisymposium on "Geometry, Statistics and Image Analysis". Dr. Paige is actively involved in undergraduate statistics education at Texas Tech University.

Dr. Padmanabhan Seshaiyer is an Associate Professor of Mathematics and Statistics at Texas Tech University. Dr. Seshaiyer's research is in the broad area of computational mathematics, with a focus on systematically developing and combining sophisticated numerical techniques with high performance computing and applying them to several computationally challenging problems arising in different applications. His research is highly interdisciplinary and it fosters involvement from high school, undergraduate and graduate students. He is the principal organiser of the 2003 Redraider Minisymposium on "Mathematical and Computational Modeling of Biological Systems". Dr. Seshaiyer is also the director of the Texas Tech Summer Mathematics Academy that exposes talented students and their teachers to the elegance of advanced mathematics and also enhances their experience in problem-solving and open ended explorations. He is also one of the organisers of the Emmy Noether Highschool Day for Women.

Dr. Magdalena D. Toda is an Assistant Professor at Texas Tech University. Her main research interests are in differential geometry and related integrable systems. Her dissertation and recent works emphasised applications of moving frame methods and Lie group theory to specific surface constructions. Her current research areas include Riemannian spaces, relativistic spaces, and geometric solutions of partial differential equations. Dr. Toda's expertise in these areas has helped undergraduate and graduate students construct educational models using the Mini-Lab and the CMC-Lab created at GANG-Amherst. Dr. Toda is preoccupied in increasing the involvement of women and underrepresented minorities in mathematics. In this respect, Dr. Toda is involved in the "Joy of Thinking" which is an after school extra-curricular mathematics program for female K-12 students.

APPENDIX

CALCULATOR EXERCISE

Course Number:---------------------------------

Consider finding the limit:

$$L = \lim_{N \to \infty} \left(1 + \frac{1}{N}\right)^N$$

Perform the following experiment to determine the limit L.

(a) Use your calculator and complete the following table.

N	$\left(1 + \frac{1}{N}\right)^N$
10^2	
10^4	
10^6	
10^8	
10^{10}	
10^{12}	
10^{14}	
10^{16}	
10^{20}	

(b) Write down the name and brand of the calculator you used to perform your calculations.

(c) Explain your observations from the experiments in part (a). For large N (i.e. as $N \to \infty$), what is the limit L is converging to? Is this the expected value? If so, why? If not, why?