Numerical Analysis (Due: November 13, 2017)
Math 685/OR 682
SOLUTION TO NONLINEAR EQUATIONS

Computer Project

Instructions: This project is worth 15 % of your total grade in the class. Your final program and related outputs must be handed in on time to receive full credit.

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1. (75 Points) Write an MATLAB program that combines the Newtons, Horners and Mullers algorithms to obtain **all** the roots of the following polynomials.

   (I) $\boldsymbol{P(x) = x^4 - 1}$

   (II) $\boldsymbol{P(x) = x^5 - 5x^4 - 8x^3 + 40x^2 - 9x + 45}$

   (III) $\boldsymbol{P(x) = 1.1x^5 - 5x^4 - 8x^3 + 40x^2 - 9x + 45}$

   Use the following steps in your implementation.

   (A) Do the following until **all** the approximate roots of $\boldsymbol{P(x)}$ are determined

      (a) Employ the Mullers algorithm to determine a root $\boldsymbol{z}$ of $\boldsymbol{P(x)}$

      (b) Deflate $\boldsymbol{P(x) = (x - z)Q(x)}$ using the Horners algorithm

      (c) Set $\boldsymbol{P(x)} = \boldsymbol{Q(x)}$.

      (d) Go to Step (a).

   (B) Use **each** approximate root obtained as an initial guess and apply the Newtons algorithm to obtain better approximation for the roots of the original polynomial.

   (C) Explain your observation for the answers obtained for (II) and (III).

2. (75 points) Consider the fixed-point method to solve for the positive real root of the polynomial $20x^3 + x^2 - 36 = 0$ using the iteration function

$$g(x) = \sqrt{\frac{36}{20x + 1}}$$

that satisfies $x = g(x)$.

(A) Find the positive real root of $20x^3 + x^2 - 36 = 0$. Show your work.

(B) Evaluate the approximate root by using the fixed-point method (contraction) algorithm with an initial guess $x_0 = 1$ that is accurate upto 10^{-6}. Using the first 10 iterations of the approximate solution $x_1, x_2, \ldots, x_{10}$, study the convergence of the method employed.

(C) Define the $\nabla x_i = x_{i+1} - x_i$ for $i \geq 0$ and $\Delta x_i = x_{i+2} - 2x_{i+1} + x_i$ for $i \geq 0$. Write a MATLAB program to implement the following steps:

(a) Using the fixed-point algorithm with $x_0 = 1$, determine $x_1 = g(x_0)$ and $x_2 = g(x_1)$. Use the three values x_0, x_1, x_2 to find $\hat{x}$ as follows:

$$\hat{x} = x_0 - \frac{(\nabla x_0)^2}{\Delta x_0}$$

Call $S_1 = \hat{x}$ for this first approximation.

(b) Using the fixed-point algorithm with $x_0 = \hat{x}$, and repeat the previous step. That is, determine $x_1 = g(x_0)$ and $x_2 = g(x_1)$. Use the three values x_0, x_1, x_2 to find $\hat{x}$ as follows:

$$\hat{x} = x_0 - \frac{(\nabla x_0)^2}{\Delta x_0}$$

Call $S_2 = \hat{x}$ for this second approximation.

(c) Perform 10 iterations of this process to obtain $S_1, S_2, \ldots, S_{10}$ and use this sequence to study the convergence of method employed here.

(D) Compare the convergence in steps (B) and (C).