

Problem 10. (10 pts) $f(x, y, z) = 2x^2 + y^2 + z^2 - xy$.

(a) Find the equation of a plane tangent to the level-surface $\{f(x, y, z) = 3\}$ at the point $P(1, 1, 1)$.

$$\vec{\nabla} f = \langle 4x - y, 2y - x, 2z \rangle \stackrel{\text{at } P}{=} \langle 3, 1, 2 \rangle$$

$$3(x-1) + 1(y-1) + 2(z-1) = 0.$$

$$\text{OR } 3x + y + 2z = 6$$

(b) Set the gradient of f parallel to \vec{j} and find the two points on the the ellipsoid $\{f(x, y, z) = 3\}$ where the tangent plane is parallel to the xz -plane.

$$\vec{\nabla} f = k\vec{j} = \langle 0, k, 0 \rangle.$$

$$\begin{cases} 4x - y = 0 \Rightarrow y = 4x. \\ 2y - x = k \Rightarrow 8x - x = k \Rightarrow x = \frac{k}{7} \ \& \ y = \frac{4k}{7} \\ 2z = 0. \Rightarrow z = 0 \end{cases}$$

$$\text{ALSO } 2\frac{k^2}{49} + \frac{16k^2}{49} - \frac{4k^2}{49} = \frac{14k^2}{49} = \frac{2k^2}{7} = 3$$

$$\Rightarrow k^2 = \frac{21}{2} \Rightarrow k = \pm \sqrt{\frac{21}{2}}$$

$$P_1 = \left(\sqrt{\frac{3}{14}}, 4\sqrt{\frac{3}{14}}, 0 \right) \ \& \ P_2 = \left(-\sqrt{\frac{3}{14}}, -4\sqrt{\frac{3}{14}}, 0 \right)$$