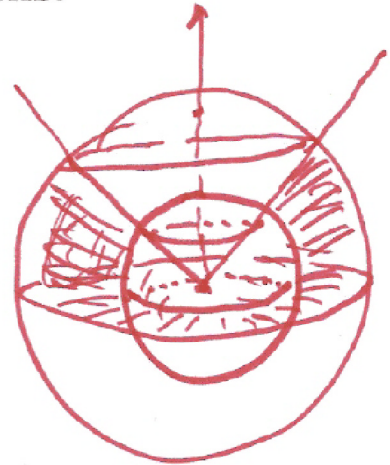


Problem 9. (10 pts) Use spherical coordinates to compute the volume of the region between two spheres centered at the origin of radius 1 and 2, above the xy -plane, and below the cone of opening $\pi/4$ about the z -axis.

$$1 \leq \rho \leq 2$$

$$0 \leq \theta \leq 2\pi$$

$$\frac{\pi}{4} \leq \varphi \leq \frac{\pi}{2}$$



$$V = \int_0^{2\pi} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_1^2 \rho^2 \sin \varphi \, d\rho \, d\varphi \, d\theta$$

$$= 2\pi \left. \frac{\rho^3}{3} \right|_1^2 (-\cos \varphi) \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= 2\pi \frac{8-1}{3} \left(0 - \left(-\frac{1}{\sqrt{2}}\right) \right)$$

$$= \boxed{\frac{14\pi}{3\sqrt{2}}}$$