Problem 3. (10 pts) Find the minimum value of

$$f(x, y) = x^2 + y^2 - xy - x + 2y + 1$$

A. $(-1, 0)$
B. 1
[✓ C.] 0
D. -1
E. None of the above

$$\nabla f = \langle 2x - y - 1, 2y - x + 2 \rangle$$

$$\begin{cases} 2x - y - 1 = 0 & \Rightarrow y = 2x - 1 \\ 2y - x + 2 = 0 & \Rightarrow 4x - 2 - x + 2 = 0 \end{cases}$$

$$\Rightarrow 3x = 0 \Rightarrow x = 0$$

$$\Rightarrow y = -1$$

$$H = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\left.\begin{array}{l} \det H = 4 - 1 = 3 > 0 \\ \& \ \partial_x^2 f > 0 \end{array}\right\} \Rightarrow (0, -1) \text{ is A LOCAL MIN.}$$

SINCE $f \to +\infty$ AS $r \to \infty$, $(0, -1)$ IS A GLOBAL MIN.

$\& \ f(0, -1) = 1 - 2 + 1 = \boxed{0}$

Problem 4. (10 pts) $f(x, y) = x \ln(xy)$. Use differentials and $f(1, 1)$ to approximate $f(0.9, 1.2)$:

[✓ A.] 0.1
B. -0.1
C. 0.2
D. -0.2
E. None of the above

$$\nabla f = \left\langle \ln(xy) + 1, \frac{x}{y} \right\rangle$$

$$\nabla f(1,1) = \langle 1, 1 \rangle \quad \& \quad f(1,1) = 0$$

$$f(x, y) \cong f(1, 1) + \frac{\partial f}{\partial x}(1,1)(x - 1) + \frac{\partial f}{\partial y}(1,1)(y - 1)$$

$$= 0 + 1 \cdot (-0.1) + 1 \cdot (0.2)$$

$$= \boxed{0.1}$$