

Problem 1. (10 pts) Find the derivative of $f(x, y) = x \sin(xy)$ at $(1, \pi)$ in the direction of $\vec{i} + \vec{j}$.

A. $\langle 1, \pi \rangle$

B. $-\sqrt{2}\pi$

C. $2\vec{i} + \pi\vec{j}$

D. $-(1 + \pi)/\sqrt{2}$

E. None of the above

$$\nabla f = \langle \sin xy + xy \cos(xy), x^2 \cos(xy) \rangle$$

$$\text{at } (1, \pi) : \nabla f = \langle -\pi, -1 \rangle$$

$$\frac{\vec{i} + \vec{j}}{|\vec{i} + \vec{j}|} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle.$$

$$D_{\vec{i} + \vec{j}} f \Big|_{(1, \pi)} = \langle -\pi, -1 \rangle \cdot \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle = -\frac{1 + \pi}{\sqrt{2}}$$

Problem 2. (10 pts) Find the tangent vector at the point $(e, 0, 1)$ to the path $\vec{r}(t) = \langle te^t, t \ln t, t^2 \rangle$.

A. $\langle e^t, \ln t, 2t \rangle$

B. $\langle 2e, 1, 2 \rangle$

C. $\langle 2, 2, 1 \rangle$

D. $\langle e, 0, 2 \rangle$

E. None of the above

$$\text{Note } \vec{r}(1) = (e, 0, 1)$$

$$\dot{\vec{r}}(t) = \langle e^t + te^t, \ln t + 1, 2t \rangle$$

$$\text{So } \dot{\vec{r}}(1) = \langle 2e, 1, 2 \rangle$$