Problem 10. (10 pts) $f(x,y,z) = x^2 + 2y^2 + z^2 - xy$.
(a) Find the equation of a plane tangent to the level-surface $\{f(x,y,z) = 3\}$ at the point $P(1,1,1)$.

$\nabla f = \langle 2x-y,\ 4y-x,\ 2z\rangle \overset{@(1,1,1)}{=} \langle 1,3,2\rangle$

$\boxed{x-1+3(y-1)+2(z-1)=0}$

OR $x+3y+2z=6$

(b) Set the gradient of f parallel to $\vec{k}$ and find the two points on the the ellipsoid $\{f(x,y,z) = 3\}$ where the tangent plane is horizontal.

$\nabla f = c\vec{k} \Rightarrow \begin{cases} 2x-y=0 & (1) \\ 4x-y=0 & (2) \\ 2z=c & (3) \end{cases}$

(1) in (2): $4x-2x=0 \Rightarrow 2x=0 \Rightarrow x=0$

(1): $y=0$

(3): $z=\frac{c}{2}$.

$f(0,0,\frac{c}{2}) = \frac{c^2}{4}$ so $\frac{c^2}{4}=3 \Rightarrow c^2=12$.
$\Rightarrow c=\pm\sqrt{12}$.

POINTS ARE $(0,0,\pm\sqrt{12})$