

Problem 9. (10 pts) Find all local extrema, classify them (loc. max., loc. min., or saddle), and evaluate  $f$  at these points for  $f(x, y) = x^3 + y^3 - 3xy + 15$ .

$$\vec{\nabla} f = \langle 3x^2 - 3y, 3y^2 - 3x \rangle$$

$$\text{Set } \vec{\nabla} f = \vec{0} \Rightarrow \begin{cases} 3x^2 - 3y = 0 \\ 3y^2 - 3x = 0 \end{cases} \Rightarrow \begin{cases} x^2 - y = 0 & (1) \\ y^2 - x = 0 & (2) \end{cases}$$

$$(1) \text{ in } (2): (x^2)^2 - x = 0 \Rightarrow x^4 - x = 0$$

$$\Rightarrow x(x^3 - 1) = 0 \Rightarrow \begin{cases} x = 0 \\ x = 1 \end{cases} \Rightarrow \begin{cases} y = 0 \\ y = 1 \end{cases}$$

$$(i) \quad x = 0 \Rightarrow y = 0 \quad \& \quad x = 1 \Rightarrow y = 1$$

$$\text{So } \vec{\nabla} f = \vec{0} \Rightarrow (0, 0) \quad \& \quad (1, 1)$$

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$$H = \begin{bmatrix} \partial_x^2 f & \partial_x \partial_y f \\ \partial_x \partial_y f & \partial_y^2 f \end{bmatrix} = \begin{bmatrix} 6x & -3 \\ -3 & 6y \end{bmatrix}; \det H = 36xy - 9$$

At (0, 0):  $\det H < 0 \Rightarrow (0, 0)$  is a SADDLE &  $f(0, 0) = 15$

At (1, 1):  $\det H > 0$  &  $6x > 0 \Rightarrow (1, 1)$  is a LOCAL MINIMUM &  $f(1, 1) = 14$