

Problem 8. (10 pts)  $f(x, y) = x^2 e^{-2y}$  and  $P = (1, 0)$ .

(a) Find the derivative of  $f$  at  $P$  in the direction of  $\vec{v} = \vec{i} + \vec{j}$ .

$$\vec{\nabla} f = \langle 2x e^{-2y}, -2x^2 e^{-2y} \rangle \stackrel{2(1,0)}{=} \langle 2, -2 \rangle$$

$$\vec{\nabla} f \cdot \frac{\vec{v}}{|\vec{v}|} = \frac{2-2}{|\vec{v}|} = \boxed{0}$$

(b) Also find the direction in which  $f$  increases most rapidly.

$$|\vec{\nabla} f| = \sqrt{8} = 2\sqrt{2}$$

$f$  increases most rapidly in the direction

$$\text{of } \boxed{\left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle}.$$