

Problem 5. (10 pts) $w = \ln(x^2 + y^2 + z^2)$, $x = \cos t$, $y = \sin t$, $z = \sqrt{t}$. Find $\frac{dw}{dt}$ at $t = 3$.

- A. $1/4$
- B. $2\sqrt{3}$
- C. $1/(2\sqrt{3})$
- D. $\cos(3) \sin(3)/\sqrt{3}$
- E. None of the above

$$\frac{\partial w}{\partial x} = \frac{2x}{x^2+y^2+z^2} \stackrel{t=3}{=} \frac{2\cos 3}{4} = \frac{\cos 3}{2}$$

$$\frac{\partial w}{\partial y} = \frac{2y}{x^2+y^2+z^2} \stackrel{t=3}{=} \frac{2\sin 3}{4} = \frac{\sin 3}{2}$$

$$\frac{\partial w}{\partial z} = \frac{2z}{x^2+y^2+z^2} \stackrel{t=3}{=} \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

$$\frac{dx}{dt} = -\sin t \stackrel{t=3}{=} -\sin 3$$

$$\frac{dy}{dt} = \cos t \stackrel{t=3}{=} \cos 3$$

$$\frac{dz}{dt} = \frac{1}{2\sqrt{t}} \stackrel{t=3}{=} \frac{1}{2\sqrt{3}}$$

$$\begin{aligned} \frac{dw}{dt} &= \frac{\partial w}{\partial x} \frac{dx}{dt} + \frac{\partial w}{\partial y} \frac{dy}{dt} + \frac{\partial w}{\partial z} \frac{dz}{dt} \\ &= -\frac{\sin 3 \cos 3}{2} + \frac{\sin 3 \cos 3}{2} + \frac{\sqrt{3}}{2} \cdot \frac{1}{2\sqrt{3}} = \frac{1}{4} \end{aligned}$$

Problem 6. (10 pts) $f(x, y) = \cos(xy)$. Also x and y are functions of u and v with $x(-1, 1) = \sqrt{\pi}$, $y(-1, 1) = \sqrt{\pi}/3$, $\frac{\partial x}{\partial u}(-1, 1) = 6\sqrt{\pi}$, $\frac{\partial x}{\partial v}(-1, 1) = -2\sqrt{\pi}$, $\frac{\partial y}{\partial u}(-1, 1) = 2\sqrt{\pi}$, $\frac{\partial y}{\partial v}(-1, 1) = 5\sqrt{\pi}$. Find $\frac{\partial f}{\partial u}$ when $(u, v) = (-1, 1)$.

- A. $-2\pi\sqrt{3}$
- B. $-7\pi\sqrt{3}/2$
- C. $7\pi\sqrt{3}/2$
- D. $-11\pi\sqrt{3}/2$
- E. None of the above

$$\frac{\partial f}{\partial x} = -y \sin(xy) \stackrel{(u,v)=(-1,1)}{=} -\frac{\sqrt{\pi}}{3} \sin \frac{\pi}{3} = -\frac{\sqrt{\pi}\sqrt{3}}{6}$$

$$\frac{\partial f}{\partial y} = -x \sin(xy) \stackrel{(u,v)=(-1,1)}{=} -\sqrt{\pi} \sin \frac{\pi}{3} = -\frac{\sqrt{\pi}\sqrt{3}}{2}$$

$$\begin{aligned} \frac{\partial f}{\partial u} &= \frac{\partial f}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial u} \stackrel{(u,v)=(-1,1)}{=} \left(-\frac{\sqrt{\pi}}{6} \cdot 6\sqrt{\pi} - \frac{\sqrt{\pi}}{2} \cdot 2\sqrt{\pi} \right) \sqrt{3} \\ &= (-\pi - \pi) \sqrt{3} = -2\pi\sqrt{3} \end{aligned}$$