Problem 8. (10 pts) Compute $\int\int_S \operatorname{Curl}\vec{F}\cdot d\vec{S}$ where S is the upper hemisphere $\{(x,y,z) : x^2+y^2+z^2=1, z>0\}$ oriented by the normals that have positive dot product with $\vec{k}$ and $\vec{F}=-y\vec{i}+x\vec{j}$.

BY STOKES' THM THERE IS SURFACE INDEPENDENCE

SO WE CAN REPLACE $\mathcal{S}$ BY THE DISK

$D^+ = \{ z=0 : x^2+y^2 \leq 1 \}$ ORIENTED UP.

i.e. $\iint_{\mathcal{S}} \operatorname{Curl}\vec{F}\cdot d\vec{S} = \iint_{D^+} \operatorname{Curl}\vec{F}\cdot d\vec{S}$

$$= \iint_{D^+} \operatorname{Curl}\vec{F}\cdot\vec{k}\, dS$$

WE NEED THE Z - COMPONENT OF $\operatorname{Curl}\vec{F} = \vec{\nabla}\times\vec{F}$

$$\langle *, *, 2\rangle =$$

$$= \langle \partial_x, \partial_y, \partial_z\rangle$$

$$\times \langle -y, x, 0\rangle$$

So $\iint_{\mathcal{S}} \operatorname{Curl}\vec{F}\cdot d\vec{S} = \iint_{D^+} 2\, dS = 2\,\text{Area}(D^+)$

$$= \boxed{2\pi}$$