

Problem 5. (10 pts) Evaluate $\int \int_D (1 + x^2 + y^2)^2 dy dx$ where D is the disk of radius 1 centered at the origin.

- A. $2\pi/5$
- B. $5\pi/7$
- C. $7\pi/3$
- D. $3\pi/2$
- E. None of the above

$$\int_0^{2\pi} \int_0^1 (1+r^2)^2 r dr d\theta$$

$$= \pi \left. \frac{(1+r^2)^3}{3} \right|_0^1 = \pi \frac{8}{3} - \frac{\pi}{3} = \frac{7\pi}{3}$$

Problem 6. (10 pts) Find $\int_C \vec{F} \cdot d\vec{r}$ where C is a straight segment from $(0,0)$ to $(1,\pi)$ and $\vec{F}(x,y) = (e^x \cos y)\vec{i} - (e^x \sin y)\vec{j}$.

- A. $-e + 1$
- B. $-e - 1$
- C. $e - 1$
- D. $e + 1$
- E. None of the above

$$(1) \int \frac{\partial f}{\partial x} = e^x \cos y \Rightarrow f = e^x \cos y + c(y)$$

$$(2) \int \frac{\partial f}{\partial y} = -e^x \sin y$$

$$\text{In (2): } -e^x \sin y + c'(y) = -e^x \sin y \Rightarrow c'(y) = 0$$

$$\Rightarrow c(y) = C$$

$$\text{So } \vec{F} = \nabla f \text{ WHERE } f(x,y) = e^x \cos y$$

$$\text{By Fund. Thm. of Calc: } \int_C \nabla f \cdot d\vec{r} = f(1,\pi) - f(0,0)$$

$$= e \cdot (-1) - e^0 \cdot 1$$

$$= -e - 1$$