Problem 3. (10 pts) Find a point of local maximum for

$$f(x,y) = x^3 + y^3 + 3x^2 - 3y^2 - 8$$

A. $(0,2)$
B. $(-2,0)$
C. $(0,0)$
D. $(-2,2)$
E. None of the above

$\nabla f = \langle 3x^2+6x,\ 3y^2-6y \rangle$

$\begin{cases} 3x(x+2)=0 \Rightarrow x=0 \text{ OR } -2 \\ 3y(y-2)=0 \Rightarrow y=0 \text{ OR } 2 \end{cases}$

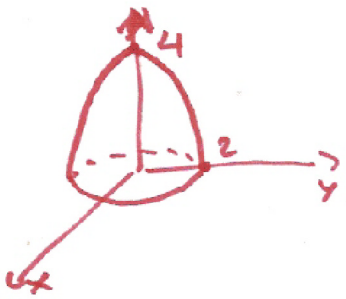
$\text{HESSIAN} = \begin{bmatrix} 6x+6 & 0 \\ 0 & 6y-6 \end{bmatrix}$ $\quad \det H = 36(x+1)(y-1)$

$\Rightarrow (0,0)$ & $(-2,2)$ ARE SADDLES

FOR $(0,2)$ $\det H > 0$ BUT $f_{xx} = 6 > 0$

Problem 4. (10 pts) Compute the volume of the truncated upside down paraboloid $E = \{(x,y,z) : 0 \le z \le 4 - x^2 - y^2\}$

A. 2π
B. 4π
C. 6π
D. 8π
E. None of the above



$$\int_0^{2\pi}\int_0^{2}\int_0^{4-r^2} r\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^{2} 4r - r^3\,dr\,d\theta$$

$$= 2\pi\left(2r^2 - \frac{r^4}{4}\right)\Big|_0^2 = 2\pi(8-4) = 8\pi$$