Math 106
Test 1, Chapter 8

Name ANSWER KEY A
February 17, 2009

Show all work neatly. You may use a calculator on this test. Put your answers on the lines provided or in boxes Unless otherwise specified, round final answers to two decimal places.

10 pts.

1. Suppose that you have $10,000 in a rather risky investment recommended by your financial advisor. During the first year, your investment increased by 35% of its original value. During the second year, your investment decreased by 30% of its first-year value. Your advisor tells you that there must have been a 5% overall increase in your original $10,000 investment. Is your advisor using percentages properly? NO If not, what is your actual percent gain or loss on your original $10,000 investment? -5.5% (or 5.5% loss) Clearly show all steps of your reasoning.

10,000 × .35 = 3500. At end of year 1, value was $10,000 + $3500 = $13,500

13500 × .30 = $4050. At end of year 2, value was $13500 − 4050 = $9450.

$\% \text{ change} = \frac{9450-10000}{10000} = \frac{-550}{10000} = -.055 = -5.5\%$

8 pts.

2. According to the 2005 *Forbes Billionaire list*, Bill Gates' net worth was $46.5 billion. Express his net worth in scientific notation, where 1 billion = 1,000,000,000.

$46.5 \times 10^9 = 4.65 \times 10 \times 10^9 = \$4.65 \times 10^{10}$

According to the same list, Warren Buffett's net worth was $44.0 billion. By how much did Bill Gates' net worth exceed that of Warren Buffett? Express your answer in scientific notation. $\$2.5 \times 10^9$

Bill Gates: 46.5×10^9
Warren Buff: 44.0×10^9
$\$2.5 \times 10^9$

12 pts.

3. At the time of her grandson's birth, a grandmother wished to deposit $5,000 in a savings account, to be given to her grandson on his 21st birthday (that is, after 21 years).
   a) Which investment should she choose for her $5,000? 4.08% cont. comp. Calculate the future value of each after 21 years.
      i) 4.1% compounded annually $11,626.13 $5000(1+.041)^{21} = 11,626.134$
      ii) 4.09% compounded semiannually $11,700.99 $5000\left(1+\frac{.0409}{2}\right)^{2\cdot 21} = \$11,700.991$
      iii) 4.08% compounded continuously $11,778.05 $5000 \cdot e^{.0408 \times 21} = \$11,778.05$

   b) What is the Effective Annual Yield of the best investment above? 4.16%

   $e^{.0408} - 1 \approx 1.04164 = .04164 \approx 4.16\%$

9 pts.

present value →

4. How much money should be deposited today in an account that pays 2.3% compounded monthly so that it will accumulate to $12,000 in three years? $11,200.66
How much interest will the account have earned in three years? $799.34

$\frac{12000}{\left(1+\frac{.023}{12}\right)^{12\cdot 3}} = \$11,200.659$

$12000 − A
− 11200.66 − P
$799.34 I

9 pts.

5. How much money should be deposited in an account at the end of every month if the account pays 2.3% compounded monthly so that it will accumulate to \$12,000 in three years? \$322.29 How much interest will the account have earned in three years? \$397.56

$$pmt = \frac{12{,}000 \cdot \left(\frac{.023}{12}\right)}{\left[\left(1+\frac{.023}{12}\right)^{12\cdot 3} - 1\right]} = 322.284\uparrow$$

Total pmts: $322.29 \times 12 \times 3 = 11{,}602.44$

$I = A - P = 12000 - 11{,}602.44 = \$397.56$

9 pts.

6. At age 25, to save for retirement, you decide to deposit \$40 at the end of each month in an IRA that pays 3.5% compounded monthly.
   a) How much will you have in the IRA account when you retire at age 65?
   \$41,786.68 $A = \frac{40 \cdot [(1+.035/12)^{12\cdot 40} - 1]}{(.035/12)} = \$41{,}786.675$
   b) Find the total interest amount in the account. \$22,586.68
   Total pmts: $\$40 \times 12 \times 40 = 19{,}200$ \$41,786.68 − 19,200 = \$22,586.68

10 pts.

7. A credit card has an annual interest rate of 18%. In the November 1-November 30 itemized billing, the November 1 unpaid balance is \$2300. A payment of \$250 was made on November 13. There are no purchases or cash advances in the billing period. If the credit card company uses the average daily balance method of computing interest, calculate the interest owed on the credit card for the month of November.

	Days	Balance
Nov 1 - Nov 12	12	2300
Nov 13 - Nov 30	18	2300 − 250 = \$2050

$$ADB = \frac{2300(12) + 2050(18)}{30} = \frac{64{,}500}{30} = \$2150$$

Int = $2150 \times .18 \times 1/12 = \boxed{\$32.25}$

13 pts.

8. The price of a condominium is \$220,000. The bank requires a 5% down payment. The cost of the condominium is financed with a 30-year fixed-rate mortgage at 6.3%.
   a) Find the required down payment. \$11,000
   b) Find the amount of the mortgage. \$209,000 = \$220,000 − 11,000
   c) Find the monthly mortgage payment. \$1293.66
   $\frac{209{,}000\left(\frac{.063}{12}\right)}{\left(1-\left(1+\frac{.063}{12}\right)^{-12\cdot 30}\right)}$ 1097.25 $1293.653\uparrow$
   d) Find the total cost of interest over 30 years. \$256,717.60
   $1293.66 \times 12 \times 30 =$ 465,717.60 − 209,000

12 pts. 9. You borrow \$18,000 for four years at 7.5% toward the purchase of a car. Your monthly payment to amortize the debt is \$435.23.
   a) Find the total cost of interest over the four years. \$2891.04
   $435.23 \times 12 \times 4 = 20891.04$ 20891.04 − 18000 = \$2891.04
   b) Prepare a loan amortization schedule for the first two months of the car loan. Round entries to the nearest cent.

Payment Number	Interest	Principal	Loan Balance
1	18000 · .075 · 1/12 = $\boxed{112.50}$	435.23 − 112.50 = $\boxed{322.73}$	18000 − 322.73 = $\boxed{17677.27}$
2	17677.27 × .075 × 1/12 = $\boxed{110.48}$	435.23 − 110.48 = 324.75	17677.27 − 324.75 = $\boxed{17352.52}$

Math 106
Test 1, Chapter 8

Name ANSWER KEY B
February 17, 2009

Show all work neatly. You may use a calculator on this test. Put your answers on the lines provided or in $\boxed{\text{boxes}}$ Unless otherwise specified, round final answers to two decimal places.

10 pts.

1. Suppose that you have \$15,000 in a rather risky investment recommended by your financial advisor. During the first year, your investment increased by 40% of its original value. During the second year, your investment decreased by 35% of its first-year value. Your advisor tells you that there must have been a 5% overall increase in your original \$15,000 investment. Is your advisor using percentages properly? NO! If not, what is your actual percent gain or loss on your original \$15,000 investment? -9% (or 9% loss) Clearly show all steps of your reasoning.

year 1: \$15,000 · .40 = \$6000 , Value @ end: \$15,000 + 6000 = \$21,000

year 2: \$21,000 · .35 = \$7350 , Value @ end: \$21,000 − 7350 = \$13,650

% change: $\frac{13,650 - 15000}{15000} = \frac{-1350}{15000} = -.09$

8 pts.

2. According to the 2005 *Forbes Billionaire list*, Paul Allen's net worth was \$21.0 billion. Express his net worth in scientific notation, where 1 billion = 1,000,000,000. $= 10^9$

$21.0 \times 10^9 = 2.10 \times 10 \times 10^9 = \$2.10 \times 10^{10}$

According to the same list, S. Robson Walton's net worth was \$18.3 billion. By how much did Paul Allen's net worth exceed that of S. Robson Walton? Express your answer in scientific notation. $\$2.[illegible] \times 10^9$

S. Robson Walton: $\$18.3 \times 10^9 = 1.83 \times 10 \times 10^9 = \$1.83 \times 10^{10}$

$$\begin{array}{r} 2.10 \times 10^{10} \\ - 1.83 \times 10^{10} \\ \hline .27 \times 10^{10} \end{array} = \$2.7 \times 10^9$$

18

3. At the time of her grandson's birth, a grandmother wished to deposit \$7,000 in a savings account, to be given to her grandson on his 21st birthday (that is, after 21 years).
   a) Which investment should she choose for her \$7,000? 3.08% compounded continuously Calculate the future value of each after 21 years.
      i) 3.1% compounded annually \$13,290.15 $\quad 7000 \times (1 + .031)^{21} = 13,290.15$
      ii) 3.09% compounded quarterly \$13,360.69 $\quad 7000\left(1 + \frac{.0309}{4}\right)^{4 \cdot 21} = 13,360.687$
      iii) 3.08% compounded continuously \$13,365.95 $\quad 7000 \cdot e^{(.0308 \cdot 21)} = 13,365.9[illegible]$

   b) What is the Effective Annual Yield of the best investment above? 3.13%

$e^{.0308} - 1 = 1.03127 - 1 = .03127 \approx 3.13\%$

9

4. How much money should be deposited today in an account that pays 2.8% compounded monthly so that it will accumulate to \$18,000 in three years? \$16,551.39

   Present Value

   How much interest will the account have earned in three years? \$[illegible]8.61

$\frac{A}{(1+\frac{r}{n})^{nt}} = \frac{18000}{(1 + \frac{.028}{12})^{12 \cdot 3}} = \$16551.382$

$$\begin{array}{r} \$18,000 = A \\ - 16,551.39 = P \\ \hline \$1,448.61 = I \end{array}$$

45

Annuity

9 pts 5. How much money should be deposited in an account at the end of every month if the account pays 2.8% compounded monthly so that it will accumulate to $18,000 in three years? $479.88 How much interest will the account have earned in three years? $724.32

$479.88 \times 12 \times 3 = 17{,}275.68$ $\quad \dfrac{18000 \cdot \frac{.028}{12}}{\left(\left(1+\frac{.028}{12}\right)^{12\cdot 3}-1\right)} = \dfrac{42}{.0875224769} = 479.876\uparrow$

$18000 - \$17{,}275.68 = \$724.32$

annuity

9 pts 6. At age 25, to save for retirement, you decide to deposit $60 at the end of each month in an IRA that pays 3.3% compounded monthly.

a) How much will you have in the IRA account when you retire at age 65?

$\boxed{\$59{,}708.63}$ $\quad A = \dfrac{60\left[\left(1+\frac{.033}{12}\right)^{12\cdot 40}-1\right]}{\frac{.033}{12}} = \dfrac{164.198...}{.00275} = 59708.632\downarrow$

b) Find the total interest amount in the account. $\boxed{\$30{,}908.63}$

$60 \times 12 \times 40 = \$28{,}800$

$I = \$59{,}708.63 - 28{,}800 = \$30{,}908.63$

10 pts 7. A credit card has an annual interest rate of 18%. In the November 1-November 30 itemized billing, the November 1 unpaid balance is $1300. A payment of $150 was made on November 14. There are no purchases or cash advances in the billing period. If the credit card company uses the average daily balance method of computing interest, calculate the interest owed on the credit card for the month of November.

Nov 1 - Nov 13: $1300, 13 days
Nov 14 - Nov 30: $1150, 17 days

$\dfrac{1300 \times 13 + 1150 \times 17}{30} = \$1215 = ADB$

Interest $= 1215 \times .18 \times \frac{1}{12} = \$18.225\uparrow = \boxed{\$18.23}$

13 pts 8. The price of a condominium is $210,000. The bank requires a 5% down payment. The cost of the condominium is financed with a 30-year fixed-rate mortgage at 6.4%.

a) Find the required down payment. $10,500

b) Find the amount of the mortgage. $199,500

c) Find the monthly mortgage payment. $1247.89

d) Find the total cost of interest over 30 years. $249,740.40

$\text{PMT} = \dfrac{199{,}500\left(\frac{.064}{12}\right)}{\left(1-\left(1+\frac{.064}{12}\right)^{-12\cdot 30}\right)}$

$1247.884\uparrow$

$449{,}240.40 = 1247.89 \times 12 \times 30$
$- 199{,}500$
$249{,}740.40$

12 pts 9. You borrow $21,000 for four years at 7.5% toward the purchase of a car. Your monthly payment to amortize the debt is $507.76.

a) Find the total cost of interest over the four years. $3372.48

$507.76 \times 12 \times 4 = 24{,}372.48$
$- 21{,}000$
$3{,}372.48$

b) Prepare a loan amortization schedule for the first two months of the car loan. Round entries to the nearest cent.

Payment Number	Interest	Principal	Loan Balance
1	$21{,}000 \times .075 \times \frac{1}{12} = \$131.25$	$507.76 - 131.25 = 376.51$	$21{,}000 - 376.51 = \$20623.49$
2	$20623.49 \times .075 \times \frac{1}{12} = \$128.90$	$507.76 - 128.90 = 378.86$	$20623.49 - 378.86 = 20244.63$

128.8968↑

53