

Math 106
Test 2, Chapter 2

Name ANSWER KEY
March 19, 2009

Show all work neatly. You may use a calculator and a 3x5 inch card with notes. Please turn in your note card when you turn in your test.

1. (24 points) Determine whether each of the following statements is true or false. If false, explain why, give a counter example, or make the necessary change(s) to make it a true statement.

a) F $-9 \in \{x | x \in \mathbb{N} \text{ and } x \leq 4\}$ *(-9 is not a natural number. $\mathbb{N} = \{1, 2, 3, \dots\}$)*

b) T $\{1, 3, 5\} \subseteq \{1, 2, 3, 4, 5\}$

c) T For any set B , if B is not the empty set, $\emptyset \subset B$ (\emptyset is a proper subset of every set)

d) F If $A \subseteq B$ and if $d \in B$, then d must be an element of A . *Counter example:*

*Let $A = \{1, 2, 3\}$; $B = \{1, 2, 3, 4\}$
 $A \subseteq B$, but $4 \in B$, $4 \notin A$.*

e) F For any set A , $A \cup \emptyset = \emptyset$.

$A \cap \emptyset = \emptyset$ (or $A \cup \emptyset = A$)

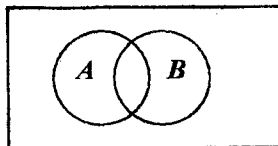
f) T The sets $A = \{x | x \in \mathbb{N} \text{ and } 1 \leq x < 6\}$ and $B = \{x | x \in \mathbb{N} \text{ and } 12 < x < 18\}$ are equivalent sets.

$A = \{1, 2, 3, 4, 5\}$ $B = \{13, 14, 15, 16, 17\}$ $n(A) = n(B)$, so they are equivalent.

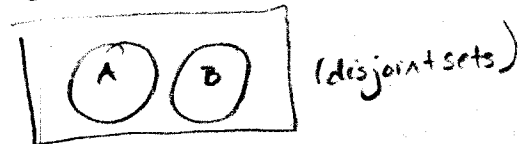
g) F The set $\{1, 2, 3, \dots, 49, 50\}$ has 50^2 subsets.

2^{50}

h) F If $A \cap B = \emptyset$, then the following Venn diagram correctly reflects the relationship between A and B .



Correct Venn is:



2. (9 points) Insert \cup or \cap as appropriate to make each of the statements below true.

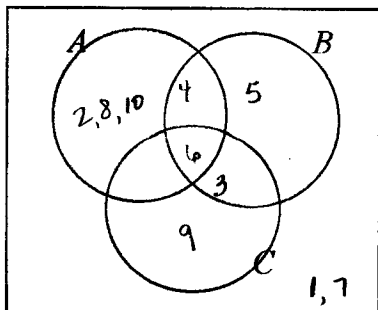
a) $\{8, 10, 12\} \cap \{4, 6, 8, 10\} = \{8, 10\}$ *(elements in common)*

b) $\{1, 3, 5\} \cup \{1, 3\} = \{1, 3, 5\}$
all of elements in either one.

c) $\{1, 3, 5\} \cap \emptyset = \emptyset$
no elements in common

3. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{2, 4, 6, 8, 10\}$, $B = \{3, 4, 5, 6\}$, and $C = \{3, 6, 9\}$.

a) (4 points) Place all elements of U in the appropriate regions of the Venn diagram below:



b) (15 points) Based on the same sets, identify, by roster, the following sets. Be sure to use correct set notation for your answers.

i) $A \cap C = \{6\}$

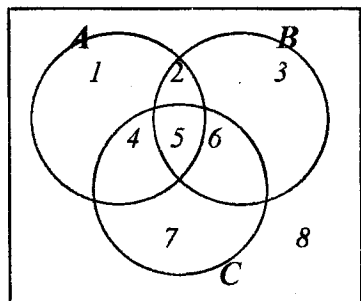
ii) $B \cup C = \{3, 4, 5, 6, 9\}$

iii) $B' \cap C = \{1, 2, 7, 8, 9, 10\} \cap \{3, 6, 9\} = \{9\}$

iv) $(A \cup B)' = \{1, 7, 9\}$

v) $A \cap B \cap C = \{6\}$

4. (16 points) Use the numbered regions in the Venn diagram below to answer the following questions.



Regions
 $A = \{1, 2, 4, 5\}$
 $B = \{2, 3, 5, 6\}$
 $C = \{4, 5, 6, 7\}$

a) Which regions represent $(A \cup B)' \cap C$? 7 (in C, outside of A or B)

b) Which regions represent $A' \cup C$? 3, 4, 5, 6, 7, 8 $A' = \{3, 6, 7, 8\}$

c) Which regions represent $(A \cap B)' \cup (B \cap A')$? 1, 3, 4, 6 $A \cap B' = \{1, 4\}$ $B \cap A' = \{3, 6\}$

d) Which regions represent $(B \cup C) \cap A$? 2, 4, 5

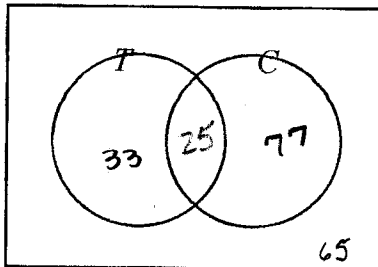
$B \cup C = \{2, 3, 4, 5, 6, 7\}$

$(B \cup C) \cap A = \{2, 4, 5\}$

↑
 in common with A.

5. (15 points) A researcher collected data on 200 households and found the following:
 58 families owned a big-screen TV (set T)
 102 families owned a digital camera (set C).
 65 families owned neither a big-screen TV nor a digital camera.

a) Complete the Venn diagram below so that it summarizes the results of this inventory:



$$65 = n(T \cup C)', \text{ so } n(T \cup C) = 200 - 65 = 135$$

$$n(T \cup C) = 135 = 58 + 102 - n(T \cap C)$$

$$135 = 160 - n(T \cap C)$$

$$n(T \cap C) = 25$$

b) Answer the following questions based on the information and Venn diagram in problem 5a above:

i) How many families owned a big-screen TV or a digital camera? $\overset{T \cup C}{\underline{135}}$

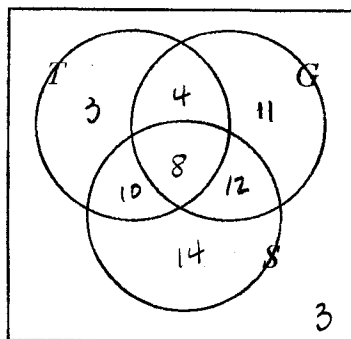
ii) How many families owned a big-screen TV but not a digital camera? $\underline{33}$

iii) How many families owned both a big-screen TV and a digital camera? $\overset{T \cap C}{\underline{25}}$

6. (17 points) After a genetics experiment, the number of pea plants having certain characteristics was tallied, with the following results:

- 25 plants were tall (set T)
- 35 had green peas (set G)
- 44 had smooth peas (set S)
- 12 were tall and had green peas $T \cap G$
- 18 were tall and had smooth peas $T \cap S$
- 20 had smooth, green peas $S \cap G$
- 8 had all three characteristics
- 3 had none of the characteristics

a) Use the Venn diagram below to reflect all of the information above.



b) How many pea plants were counted?

$$\underline{65}$$

c) How many pea plants were tall or had green peas?

$$\underline{48} \quad 3 + 4 + 11 + 10 + 8 + 12 =$$

d) How many pea plants were not tall but had peas that were smooth or green?

$$\underline{37} \quad 11 + 12 + 14 =$$