

Math 106

Homework, sections 3.1-3.3

Name ANSWER KEY

Due at beginning of class Thursday, April 2

1. Write a statement that represents the **negation** of each of the statements below. (The negation of a quantified statement should begin with "all," "some," or "no," and none of the negations should begin with "It is not true that....")

a) Frank is not a fan of "ER."

Frank is a fan of "ER."

b) No lunch is free.

Some lunches are free. (At least one lunch is free.)

c) Some elephants are pink.

No elephants are pink.

d) All colas contain caffeine.

Some colas do not contain caffeine.

e) At least one banana is not ripe.

All bananas are ripe. (No bananas are not ripe.)

2. Write each of the statements below in symbolic form. Assign letters to simple statements that are not negated.

a) I do not sleep soundly if I drink coffee or eat chocolate.

$$\boxed{(p \vee q) \rightarrow \sim r}$$

p: I drink coffee.
q: I eat chocolate
r: I sleep soundly

b) All people born in the United States are American citizens.

(person is born in the United States, that person is an American citizen.)

$$\boxed{p \rightarrow q}$$

Equivalent to: "If a person is born in the US, that person is an American citizen."
p: a person is born in the US.
q: a person is an American citizen.

c) Your check will not be accepted if you do not have a drivers' license and a credit card.

$$\boxed{\sim(p \wedge q) \rightarrow \sim r}$$

conclusion ← premise
(arrow points from premise to conclusion.)

p: you have a drivers license
q: you have a credit card
r: your check is accepted.

3. Let *p* and *q* represent the following simple statements:

p: I am innocent.
q: I have an alibi.

a) Using the simple statements above, express each of the following in words:

i) $p \wedge q$ I am innocent and have an alibi.

ii) $\sim q \rightarrow \sim p$ if I don't have an alibi, then I'm not innocent.

iii) $q \vee \sim p$ I have an alibi or I'm not innocent.

b) Using the same simple statements from question 3 above, write each of the following statements in symbolic form.

i) My having an alibi is necessary for being innocent.

$$\boxed{P \rightarrow q}$$

q is necessary for p is equivalent to $\sim p, \text{ then } q.$
(If I am innocent, I have an alibi.)

ii) I am innocent if and only if I have an alibi.

$$\boxed{P \leftrightarrow q}$$

iii) I have an alibi but I am not innocent.

$$\boxed{q \wedge \sim P}$$

iv) I will be innocent if I have an alibi.

conclusion $P \leftarrow q$ premise (arrow points from premise to conclusion)

$$\boxed{q \rightarrow P}$$

c) Construct a truth table for the statement "I have an alibi but I am not innocent," ($q \wedge \sim P$) (biii above) and indicate all conditions under which the statement is true.

1	2	3	2 \wedge 3
P	q	$\sim P$	$q \wedge \sim P$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

← only instance where the statement is true - that is, person has an alibi but is not innocent.

4. Construct a truth table for each of the following symbolic expressions:

a) $\sim p \vee q$

1	2	3	3 \vee 4
P	q	$\sim P$	$\sim P \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

(FVF: F, otherwise T)

b) $p \vee \sim(p \wedge q)$

1	2	3	4	1 \vee 4
P	q	$P \wedge q$	$\sim(P \wedge q)$	$P \vee \sim(P \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

c) $(p \vee q) \wedge (\sim p \vee \sim q)$

1	2	3	4	5	6	3 \wedge 6
P	q	$P \vee q$	$\sim P$	$\sim q$	$\sim P \vee \sim q$	$(P \vee q) \wedge (\sim P \vee \sim q)$
T	T	T	F	F	F	F
T	F	T	F	T	T	T
F	T	T	T	F	T	T
F	F	F	T	T	T	F

(TAT: T, otherwise F)