

- You may use notes and your book to help you answer the questions on this test.
- You *may not* communicate with any other person about this test.
- Any evidence that your answers are not completely your own will be dealt with as harshly as possible.
- Your answers must be neat and legible. Anything that I can't read easily will be given a grade of 0.

1. Let  $p$ ,  $q$ , and  $r$  represent the following simple statements:

$p$ : The car costs \$75,000.

$q$ : The car goes 160 miles per hour (mph).

$r$ : The car attracts police attention.

a) Using the statements above, express each of the following statements symbolically:

10  
3  
i) The car will go 160 mph if it costs \$75,000. (If the car costs \$75,000, then it will go 160 mph.)

$p \rightarrow q$

2  
ii) All cars that cost \$75,000 attract police attention. (If a car costs \$75,000, then it will attract police attention.)

$p \rightarrow r$

2 1/2  
iii) If the car does not go 160 mph and does not attract police attention, it will not cost \$75,000.

$(\sim q \wedge \sim r) \rightarrow \sim p$

2  
iv) The car goes 160 mph but it did not cost \$75,000.

$q \wedge \sim p$

2  
v) It is necessary and sufficient for a car to go 160 mph for it to attract police attention.

$q \leftrightarrow r$

b) Using the same statements, write each of the symbolic statements below in words:

9  
3  
i)  $(p \wedge q) \rightarrow r$

If the car costs \$75,000 and goes 160 mph, it will attract police attention.

3  
ii)  $\sim q \leftrightarrow \sim r$

A car does not go 160 mph if and only if it does not attract police attention. (Not going 160 mph is necessary & sufficient for not attracting police attention.)

3  
iii)  $(q \vee r) \wedge p$

The car goes 160 mph or attracts police attention, and it costs \$75,000.

2. Write a statement (in words) that represents the **negation** of each of the statements below. None of the negations should begin with "It is not true that...", and the negations of quantified statements should begin with "all," "none," "some," or "at least one."

a) No man born of a woman could kill Macbeth.

*At least one man born of a woman could kill Macbeth.*

b) Some red cars do not attract police attention.

*All red cars attract police attention.*

c) Sally is a fan of "Damages," but Jane does not like it.

*Sally is not a fan of "Damages," or Jane likes it.*

d) All cats are gray in the dark.

*Some cats are not gray in the dark.*

*p: A's on all 3 tests*

*q: B or better on 10 of 12 quizzes*

*r: completed 5 of 6 HW.*

*Exempt:  $p \wedge (q \vee r)$*

*(re must meet p,*

*then at least one*

*of q or r.*

3. In order to be exempt from the final test of the semester, a student must have earned A's on all of the prior 3 tests, and must have taken and earned B or better on 10 of 12 quizzes or have completed 5 of 6 homework assignments with a grade of 85% or better. Below are the current standings of 4 students.

- Sally earned grades of A on all 3 tests, completed 4 homework assignments with grades of 100%, and took 9 quizzes and earned grades of A on all 9.
- Fred earned grades of A on all 3 tests, earned B's on the 10 quizzes that he took and turned in no homework assignments.
- Sam earned A's on 2 tests and a B+ on the third, earned grades of B or better on all 12 quizzes and completed all homework assignments with a grade of 85% or better.
- Aurelia earned A's on all 3 tests, completed 5 homework assignments with a grade of 85% or better, and earned a C on one quiz, B's on 8 more quizzes, and did not take the rest.

Determine whether each student is or is not exempt from the last test, and briefly explain your reasoning.

- Sally Not exempt. Satisfied p, (A's on all 3 tests) but did not satisfy either q or r (must satisfy one or the other)
- Fred Exempt: satisfied p (required) and q (r is not necessary)
- Sam Not exempt: did not satisfy p.
- Aurelia Exempt: satisfied p, satisfied r. q was not necessary since she satisfied r.

4. Let  $p$  and  $q$  represent the following simple statements:  
 $p$ : Washington DC is in England.  $F$   
 $q$ : Paris is in France.  $T$

First determine whether each of the above statements is in fact true or false. Then write each of the following compound statements symbolically and use your knowledge of truth values of compound statements to determine whether the compound statement is true or false.

- a) T  $F \rightarrow F : T \quad p \rightarrow \sim q$   
 If Washington DC is in England, then Paris is not in France.
- b) T  $F \vee T : T \quad \sim q \vee \sim p$   
 Paris is not in France or Washington DC is not in England.
- c) T  $F \leftrightarrow F : T \quad F \leftrightarrow F : T$   
 Paris is not in France if and only if Washington DC is in England.
- d) F  $F \wedge T : F \quad \sim q \wedge \sim p$   
 Paris is not in France and Washington DC is not in England.

ORIGINAL

$p$	$q$	$\sim p$	$\sim q$	$p \rightarrow q$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

5. A newspaper article states that if global temperatures do not stop rising, all Arctic ice will melt. Determine whether each of the following statements is or is not logically equivalent to the statement in the newspaper and put your answer (equivalent/not equivalent) in the space provided. You will need to show work by either constructing a truth table or using other information about logical equivalence to justify each answer.

a) Not equivalent If all Arctic ice melts, then global temperatures did not stop rising. (Fallacy of converse) ( $q \rightarrow \sim p$ )

b) Equivalent Global temperatures stop rising or all Arctic ice melts. ( $p \vee q$ )

c) Not equivalent Some Arctic ice does not melt and global temperatures do not stop rising. ( $\sim q \wedge \sim p$ )

d) Equivalent If some Arctic ice does not melt, then global temperatures have stopped rising. Contrapositive ( $\sim q \rightarrow \sim p$ )

e) Not equivalent If global temperatures stop rising, then some Arctic ice will not melt. Fallacy of inverse. ( $p \rightarrow \sim q$ )  
 Fallacies are not equivalent.

$p$	$q$	$\sim q$	$\sim p$	$\sim q \wedge \sim p$
T	T	F	F	F
T	F	T	F	F
F	T	F	T	F
F	F	T	T	T

$p$	$q$	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

6. Use the standard form of valid arguments to draw a valid conclusion from the premises below or state that no valid conclusion can be drawn. Briefly justify your answer.

If the defendant is innocent, he does not go to jail.  
 The defendant goes to jail.

Therefore,..... the defendant is not innocent.

(contrapositive reasoning, a valid conclusion)

$p$ : def is innocent  
 $q$ : def goes to jail  
 $p \rightarrow \sim q$   
 $q$   
 $\therefore \sim p$