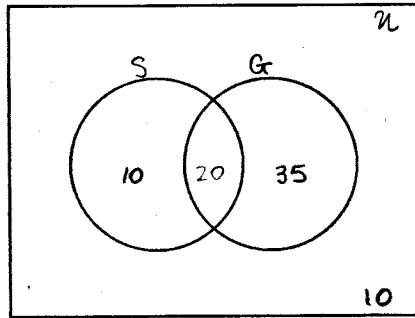


## Math 106, Practice problems on set cardinality

SPRING 2009

1. A survey of 75 students was taken, asking whether the student had slept less than 4 hours any one night in the past week and whether the student had a GPA of at least a 2.5. Of the 75 students, 30 had slept less than 4 hours in one night (set  $S$ ); 55 had a GPA of at least 2.5 (set  $G$ ), and 20 had both slept less than 4 hours and had at least a 2.5 GPA.
- a) Use the Venn diagram below and determine the cardinality of each region:



$$n(S) = 30$$

$$n(G) = 55$$

$$n(S \cap G) = 20$$

$$n(S \cup G) = n(S) + n(G) - n(S \cap G) \\ = 30 + 55 - 20 = 65$$

$$n(S \cup G)' = n(U) - n(S \cup G) \\ = 75 - 65 = 10$$

- b) How many students slept less than 4 hours or had a GPA of at least 2.5?  $n(S \cup G)$

65 students

- c) How many students did not sleep less than 4 hours and had a GPA of less than 2.5?

$$n(S \cup G)' = n(S' \cap G') = 10 \text{ students}$$

(De Morgan's Law)

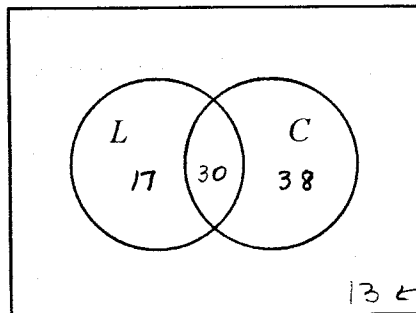
- d) How many students had a GPA of at least 2.5 and did not sleep less than 4 hours?  $n(G \cap S')$

35 students

(in  $G$ , not in  $S$ )

2. A survey of 98 students was conducted, asking whether the student lived on campus and whether the student owned a car. Of the 98 students, 47 lived on campus (Set  $L$ ); 68 owned a car (Set  $C$ ), and 13 did not live on campus and did not own a car.

- a) Use the Venn diagram below and determine the cardinality of each region:



$$n(L) = 47$$

$$n(C) = 68$$

$$n(L' \cap C') = n(L \cup C)' = 13$$

(note:  $n(U) = n(L \cup C) + n(L \cup C)'$ , so  $98 = n(L \cup C) + 13$ ;

$$n(L \cup C) = 85$$

$$\text{size of intersection: } n(L \cap C) = n(L) + n(C) - n(L \cup C) \\ = 47 + 68 - 85 = 30$$

$$13 \leftarrow n(L \cup C)' = n(L' \cap C')$$

- b) How many students lived on campus and owned a car?  $n(L \cap C) = 30$  students

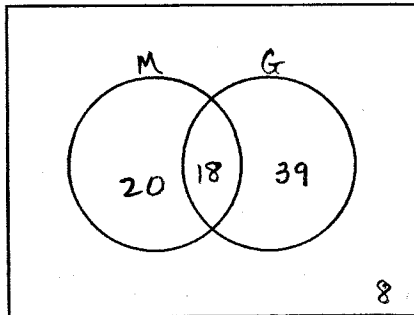
- c) How many students lived on campus but did not own a car? 17 students

- d) How many students lived on campus or owned a car? 85 students

$$n(L \cup C)$$

3. A survey of 85 students was taken, asking the questions whether the student had eaten at McDonalds in the past week and whether the student had a GPA of at least a 2.5. Of the 85 students, 38 had eaten at McDonalds; 57 had a GPA of at least 2.5; and 77 had eaten at McDonalds or had at least a 2.5 GPA. ( $n(M \cup G)$ )

a) Use the Venn diagram below and determine the cardinality of each region:



$$\begin{aligned} n(M) &= 38 \\ n(G) &= 57 \\ n(M \cup G) &= 77 \end{aligned}$$

Need to solve for intersection:  $n(M \cap G) = n(M) + n(G) - n(M \cup G)$

$$= 38 + 57 - 77 = 18 \text{ students}$$

$$8 = n(U) - n(M \cup G)$$

b) How many students did not eat at McDonalds and had a GPA of less than 2.5? (outside of both areas)  
8 students  $n(M' \cap G') = n(M \cup G)'$  (de Morgan's Law)

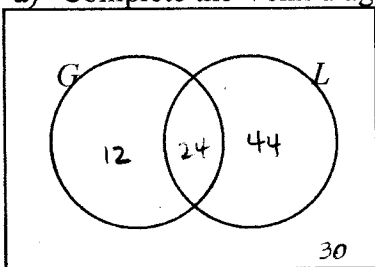
$n(M \cap G)$

c) How many students ate at McDonalds and had a GPA of at least 2.5? 18 students (intersection)

d) How many students had a GPA of at least 2.5 and did not eat at McDonalds? 39 students  
 $n(G \cap M')$  (in  $\bar{G}$ , not in  $M$ )

4. In a survey of 110 college students, 36 planned to go to graduate school immediately after college (Set  $G$ ), 68 wanted to live in the DC metropolitan area (Set  $L$ ), and 44 wanted to live in DC but not go to graduate school. ( $n(L \cap G')$ )

a) Complete the Venn diagram below so that it summarizes the results of this survey:



$$\begin{aligned} n(G) &= 36 \\ n(L) &= 68 \end{aligned}$$

All of  $L$  is 2 parts: I: in the intersection with  $G$  and II: not in the intersection.  $n(\text{not in intersection}) = 44$ .  $\Rightarrow$  part in intersection is  $68 - 44 = 24$  students.

$$n(G \cup L) = 80 = n(G) + n(L) - n(G \cap L) = 36 + 68 - 24 = 80$$

b) How many students planned to go to graduate school or live in the DC area? 80 students  
 $n(G \cup L)$

c) How many students did not plan to go to graduate school and did not want to stay in the DC area? 30 students  
 $n(G' \cap L') = n(G \cup L)'$

d) How many students planned to go to graduate school in the DC area? 24 students

$$n(G \cap L) = 24$$