ANSWER KEY

Math 106 **Questions from old tests, Spring 2009**

1. Indicate whether each of the following statements is true or false. If false, explain why, give a counter example, or make the necessary change(s) to make the statement true.

a) F $\emptyset = \{0\}$ $\emptyset$ has no elements. {0} has one element (the number 0)
or $\emptyset = \{\}$

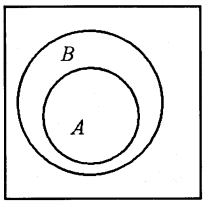
b) T If $A = B \cap C$, where A, B, and C are sets, then $A \subseteq B$.

c) F If $A \cap B = \emptyset$, then $A = B$. (if $A \cap B = \emptyset$, then A & B are disjoint sets - no elements in common.)

d) F The number of subsets of the set $\{1, 2, 3 \ldots 48, 49, 50\}$ is 2^6. # of subsets is 2^{50}

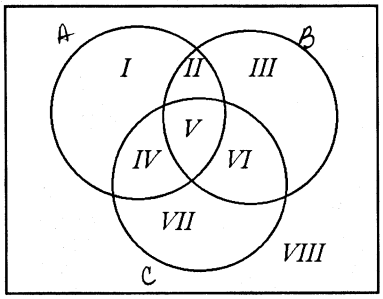
e) T If $A \subseteq B$ and $B \subseteq A$, then $A = B$.

f) F The following diagram correctly represents the relationship $B \subset A$.



← This is the relationship $A \subset B$.

2. Use the Venn diagram shown below to answer the following questions:



a) Which regions represent $A' \cap C$? VI, VII (in C, not in A)

b) Which regions represent B'? I, IV, VII, VIII

c) Which regions represent $B \cup C$? II, III, IV, V, VI, VII (6 regions)

d) Which regions represent $A \cap B \cap C$? V (1 region only)

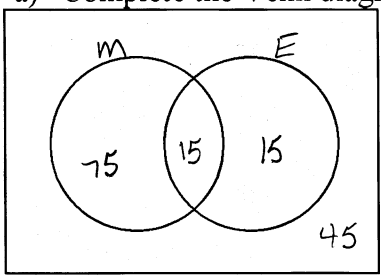
e) Which regions represent $(A \cup B \cup C)'$? VIII

f) Which regions represent $A \cup B'$? I, II, IV, V, VII, VIII
6 regions

5. In a survey of 150 college students, 90 were taking mathematics, 30 were taking economics, and 45 were taking neither mathematics nor economics.

$(M \cup E)'$

a) Complete the Venn diagram below so that it summarizes the results of this survey:



$n(M \cup E) = 150 - 45 = 105 = 90 + 30 - n(M \cap E)$

$n(M \cap E) = 120 - 105 = 15$

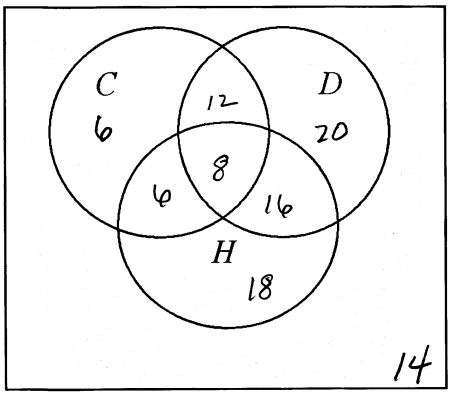
b) How many students were taking both economics and mathematics? 15 $(n(M \cap E))$

c) How many students were taking economics or mathematics? 105 $(n(M \cup E))$

d) How many students were taking economics but not mathematics? 15

6. A survey of 100 aging hippies revealed the following sad facts:
   32 color their hair (Set C)
   56 have dieted at least once in the past 6 months (Set D)
   48 have suffered hearing loss from excessive rock & roll (Set H)
   20 color their hair and have dieted
   14 color their hair and have hearing loss
   24 have suffered hearing loss and have dieted
   8 color their hair, have dieted and have suffered hearing loss

a) Use the Venn diagram below to reflect all of the information above. Then answer the questions about the data.



20
12
36
18
86

b) How many aging hippies did not color their hair, did not diet and suffered no hearing loss? 14

c) How many aging hippies colored their hair only? (no dieting or hearing loss) 6

3. Determine whether each of the following statements is true or false. If false, explain why.

a) T {June, July, August} $\subseteq$ {August, June, July} (order doesn't matter)
(sets may be equal)

b) F {June, July, August} $\subset$ {x | x is a month that begin with the letter J} August is not an element of 2nd set.

c) F {July} $\in$ {June, July, August} {July} is a set, {July} $\subseteq$ {June, July, August}

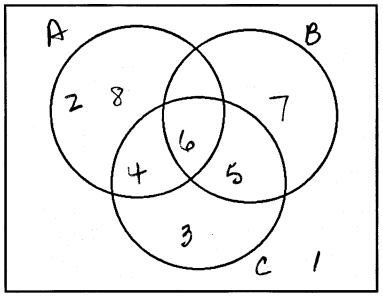
d) F $\emptyset \not\subset$ {June, July, August} $\emptyset$ is a subset of every set.

e) T July $\in$ {June, July, August}

f) F The number of *proper* subsets of {June, July, August} is 2^3. # of proper subsets is 2^3-1.

4. Let $U = \{1,2,3,4,5,6,7,8\}$, $A = \{2,4,6,8\}$, $B = \{5,6,7\}$, and $C = \{3,4,5,6\}$.

a) Place all elements of U in the appropriate regions of the Venn diagram below:



b) Based on the same sets, identify, by roster, the following sets. Be sure to use correct set notation for your answers.

i) $(A \cap B)' = \{1,2,3,4,5,7,8\} = (\{6\})'$

ii) $B' = \{1,2,3,4,8\}$

iii) $A \cap B \cap C = \{6\}$

iv) $A' \cup B' = \{1,3,5,7\} \cup \{1,2,3,4,8\} = \{1,2,3,4,5,7,8\}$

v) $B' \cup A = \{2,4,6,8,3,1\}$

vi) $(A' \cap B) \cup C = \{5,7\} \cup \{3,4,5,6\} = \{3,4,5,6,7\}$

c) What relationship, if any, is there between your answers to parts i and iv above? What might explain that outcome? Identical because of deMorgan's Laws.
$(A \cap B)' = A' \cup B'$