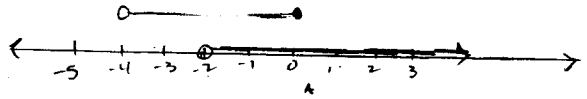
Math 105, Precalculus
Test 1, Chapter 1 and section 2.1

Name ANSWER KEY A
September 24, 2009

Show all work neatly. Use of calculators is not permitted. Please put your answers in boxes.



1. Let $A=\{x|x>-2\}$ and $B=\{x|-4<x\le 0\}$. Find the following sets:
   a) $A\cap B$ (elements in common) $\boxed{A\cap B=\{x|-2<x\le 0\} \text{ or } (-2,0]}$
   b) $A\cup B$ (elements in either) $\boxed{A\cup B=\{x|x>-4\} \text{ or } (-4,\infty)}$

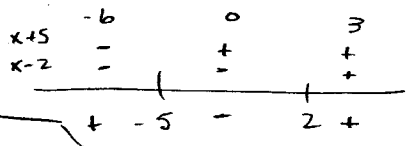
2. Factor completely:
   $4x^2-24x+36 = 4(x^2-6x+9) = 4(x-3)(x-3)$
   $= \boxed{4(x-3)^2}$

3. Solve the following equations:
   a) $\frac{x-2}{4x+7}=\frac{1}{5}$; $5(x-2)=1(4x+7)$
   $5x-10=4x+7$
   $\boxed{x=17}$

   b) $3x^2-6x-2=0$ use quadratic formula:
   $x=\frac{-(-6)\pm\sqrt{(-6)^2-4(3)(-2)}}{2\cdot 3}=\frac{6\pm\sqrt{36+24}}{6}=\boxed{\frac{6\pm\sqrt{60}}{6}}=$ fine to leave here.
   $\frac{6\pm 2\sqrt{15}}{6}=\frac{3\pm\sqrt{15}}{3}$

   c) $x^4=x^6$
   $0=x^6-x^4=x^4(x^2-1)=x^4(x+1)(x-1)$
   Let: $x^4=0$ $x+1=0$ $x-1=0$
   $\boxed{x=0 \quad x=-1 \quad x=1}$

4. Solve the following inequalities
   a) $x^2+3x-10\ge 0$ Let $(x+5)(x-2)=0$
   $(x+5)(x-2)\ge 0$ $x=-5$ $x=2$



   $\boxed{\text{Solution: } \{x|x\le -5 \text{ or } x\ge 2\} \text{ or } (-\infty,-5]\cup[2,\infty)}$

   b) $|x-3|<2 \Rightarrow$
   $-2<x-3<2$
   $+3 \quad +3 \quad +3$
   $1<x<5$; $\boxed{\text{Solution } \{x|1<x<5\} \text{ or } (1,5)}$

   c) $-2\le 4-3x<16$
   $-4 \quad -4 \quad -4$
   $-6\le -3x<12$ Divide by -3
   $2\ge x>-4$; $\boxed{-4<x\le 2 \text{ or } (-4,2]}$

10

5. Answer the questions below about the linear equation $x-3y+15=0$.

3 a) What is the slope? $m = 1/3$ $\quad x+15 = 3y$ $\quad y = 1/3\,x+5$

3 b) What is the y-intercept? $y=5$ $\quad$ or $(0,5)$

4 c) What is the equation of a line that is parallel to the line above and that passes through the point (-6, 9)? $9 = 1/3(-6)+b = -2+b$; $b=11$

$\boxed{y = 1/3\,x+11}$

6. Let $P=(-1,4)$ and let $Q=(5,2)$.

10

a) In slope-intercept form, what is the equation of the line through P and Q?

5 $m = \frac{2-4}{5-(-1)} = \frac{-2}{6} = -1/3$ $\quad y-4 = -1/3(x+1) = -1/3\,x - 1/3$

$\boxed{y = -1/3\,x + 3\,2/3 = -1/3\,x + 11/3}$

b) Which point is closer to the point $S=(3,7)$?

5 $d_1 = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$. $d_1 = \sqrt{(3+1)^2+(7-4)^2} = \sqrt{16+9} = \sqrt{25}$ - smaller,

$d_2 = \sqrt{(3-5)^2+(7-2)^2} = \sqrt{(-2)^2+5^2} = \sqrt{4+25} = \sqrt{29}$

so point P is closer

8

7. Determine whether the equation represents a circle, a point, or has no graph. If the equation is that of a circle, find its center and radius.

$x^2+y^2-8x+4y+18=0$

$x^2-8x+16+y^2+4y+4 = -18+16+4$ $\quad b=-8$ $\quad b=4$

$(x-4)^2+(y+2)^2 = 2 = r^2$ $\quad \frac{b}{2} = \frac{-8}{2} = -4$ $\quad \frac{b}{2} = 2$

$(\frac{b}{2})^2 = 16$ $\quad (\frac{b}{2})^2 = 4$

circle

Center: $(4,-2)$

Radius: $\sqrt{2}$

1

8. Find the **domain** of each of the functions.

6 a) $f(x) = \sqrt{\frac{x+1}{x-2}}$. $\quad$ let $x+1=0$ $\quad x=-1$ $\quad x-2=0$ $\quad x=2$

	-2		0		3
$x+1$	-		+		+
$x-2$	-	|	-	|	+
	+	-1	-	2	+

* Quantity under radical must be greater than or equal to 0. (+) * No zero denominator. ($x \neq 2$)

Domain: $\{x \mid x \leq -1 \text{ or } x > 2\}$ or $(-\infty,-1] \cup (2,+\infty)$

4 b) $f(z) = \frac{z-1}{z^3-6z^2-16z}$

* No zero denominator. So let $z^3-6z^2-16z=0$

Domain: $\{x \mid x \neq 0, x \neq 8, x \neq -2\}$

$z(z^2-6x-16)=0$

$z(z-8)(z+2)=0$

$z=0$ $\quad z=8$ $\quad z=-2$

7 c) $g(x) = x^2+4x-3$

Domain: $x \in \mathbb{R}$ (all reals) — No issue with denominators or radicals.

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9. If $f(x) = x^2 - 3x$, find each of the following:

2 a) $f(4) = 4^2 - 3(4) = 16 - 12 = \boxed{4}$

2 b) $f(0) = 0^2 - 3(0) = \boxed{0}$

3 c) $f(a+h)$ (expand your answer; do not leave it factored)

$= (a+h)^2 - 3(a+h) = \boxed{a^2 + 2ah + h^2 - 3a - 3h}$

$= (a+h)(a+h) - 3(a+h) =$

2 d) $f(x^2) = (x^2)^2 - 3(x^2) = \boxed{x^4 - 3x^2}$

6

10. If $f(x) = \begin{cases} 3x^2, \text{ if } x < -1 \\ 2x+1, \text{ if } x \geq -1 \end{cases}$, find each of the following:

2 a) $f(2) = 2 \cdot 2 + 1 = \boxed{5}$

2 b) $f(-2) = 3(-2)^2 = 3 \cdot 4 = \boxed{12}$

✓ c) $f(-1) = 2(-1) + 1 = \boxed{-1}$

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Math 105, Precalculus Name ANSWER KEY B

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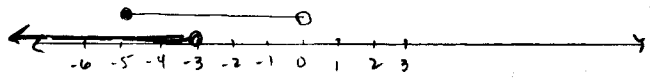


8

1. Let $A=\{x|x<-3\}$ and $B=\{x|-5\le x<0\}$. Find the following sets:

4 a) $A\cup B$ (in either set): $A\cup B=\{x\mid x<0\}$ or $(-\infty,0)$

4 b) $A\cap B$ (in both A & B, elements in common): $\{x\mid -5\le x<-3\}$ or $[-5,-3)$

2. Factor completely:

6 $2x^2-20x+50 = 2(x^2-10x+25) = 2(x-5)(x-5)$

$=\boxed{2(x-5)^2}$

3. Solve the following equations:

a) $3x^2-4x-1=0$ use quadratic formula: $x=\dfrac{4\pm\sqrt{(-4)^2-4(3)(-1)}}{2\cdot 3}=$

5

$\dfrac{4\pm\sqrt{16+12}}{6}=\boxed{\dfrac{4\pm\sqrt{28}}{6}}=$

$\dfrac{4\pm 2\sqrt{7}}{6}=\dfrac{2\pm\sqrt{7}}{3}$

b) $\dfrac{x-2}{5x+7}=\dfrac{1}{6}$

5 $(x-2)(6)=(5x+7)1$

$6x-12=5x+7$ $\boxed{x=19}$

c) $x^3=x^5$

6 $0=x^5-x^3=x^3(x^2-1)$

$=x^2(x+1)(x-1)$ → Let $x^2=0$ $x+1=0$ $x-1=0$

$\boxed{x=0 \quad x=-1 \quad x=1}$

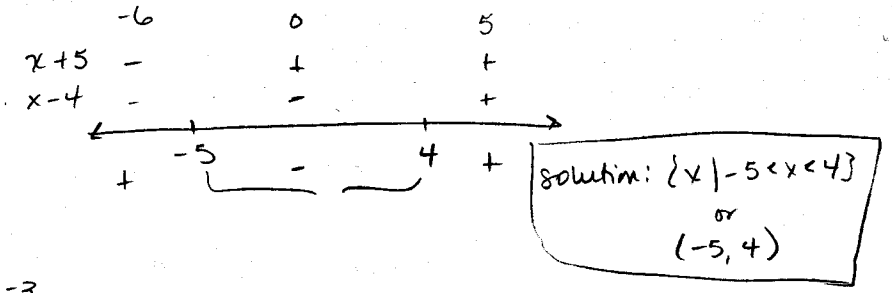
4. Solve the following inequalities

a) $x^2+x-20<0$

$(x+5)(x-4)<0$

Let $(x+5)(x-4)=0$

$x=-5$ $x=4$



solution: $\{x\mid -5<x<4\}$ or $(-5,4)$

b) $|x+4|\ge 1$

split $x+4\ge 1$ $x\ge -3$

$x+4\le -1$ $x\le -5$

solution: $\{x\mid x\le -5 \text{ or } x\ge -3\}$ or $(-\infty,-5]\cup[-3,+\infty)$

c) $-1\le 4-5x<19$

$-4 \quad -4 \quad -4$

$-5\le -5x<15$ divide by -5:

$1\ge x>-3$ or $\boxed{-3<x\le 1}$ or $\boxed{(-3,1]}$

10 pts.

5. Answer the questions below about the linear equation $x-2y+6=0$.

3 a) What is the slope? $m = 1/2$

$x+6=2y$

$y = 1/2x+3$

3 b) What is the y-intercept? $y=3$ (or $(0,3)$)

4 c) What is the equation of a line that is parallel to the line above and that passes through the point (-2, 7)? $y-7 = 1/2(x+2) = 1/2x+1$ $\boxed{y = 1/2x+8}$

6. Let $P=(-3,5)$ and let $Q=(5,3)$.

5 a) In slope-intercept form, what is the equation of the line through P and Q?

$\frac{3-5}{5-(-3)} = \frac{-2}{8} = -1/4 = m$ $\quad y-3 = -1/4(x-5) = -1/4x+5/4$

$y = -1/4x + 17/4 = -1/4x + 4\,1/4$

5 b) Which point is closer to the point $S=(2,7)$? Distance $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$d_1 = \sqrt{(2+3)^2+(7-5)^2} = \sqrt{25+4} = \sqrt{29}$

$d_2 = \sqrt{(2-5)^2+(7-3)^2} = \sqrt{(-3)^2+(4)^2} = \sqrt{9+16} = \sqrt{25}$ ← closer. ∴ Point Q is closer

7. Determine whether the equation represents a circle, a point, or has no graph. If the equation is that of a circle, find its center and radius.

$x^2+y^2+6x-2y+7=0$

$x^2+6x+9 + y^2-2y+1 = -7+9+1$

$b=6$ $\quad b=-2$

$\frac{b}{2}=3$ $\quad (\frac{b}{2})=-1$

$(\frac{b}{2})^2=9$ $\quad (\frac{b}{2})^2=1$

8 $(x+3)^2+(y-1)^2 = 3 = r^2$

circle

Center: $(-3, 1)$

radius: $\sqrt{3}$

8. Find the domain of each of the functions.

4 a) $f(x) = \dfrac{x+1}{x^3+7x^2-18x}$

Problem: No zeros allowed in denominator.

Let $x^3+7x^2-18x=0$

$x(x^2+7x-18)=0$

$x(x+9)(x-2)=0$

$x=0$; $x=-9$; $x=2$

Domain: $\{x \mid x\neq 0, x\neq -9, x\neq 2\}$

2 b) $g(x) = x^2+5x-3$

No problems. ∴ domain is $x \in \mathbb{R}$

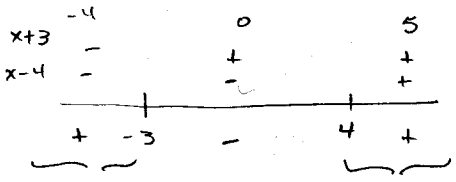
6 c) $f(x) = \sqrt{\dfrac{x+3}{x-4}}$

2 problems: * quantity under radical must be ≥ 0

* No zero denominator (∴ $x\neq 4$)

Let $x+3=0$ $\quad x-4=0$

$x=-3$ $\quad x=4$



Domain: $\{x \mid x\leq -3$ or $x>4\}$ or $(-\infty, -3] \cup (4, +\infty)$

9. If $f(x) = x^2 - 2x$, find each of the following:

a) $f(5) = 25 - 10 = \boxed{15}$

b) $f(0) = 0^2 - 2 \cdot 0 = \boxed{0}$

c) $f(x^3) = (x^3)^2 - 2(x^3) = \boxed{x^6 - 2x^3}$

d) $f(a+h)$ (expand your answer; do not leave it factored)

$= (a+h)^2 - 2(a+h)$

$= (a+h)(a+h) - 2(a+h) = \boxed{a^2 + 2ah + h^2 - 2a - 2h}$

10. If $f(x) = \begin{cases} 4x^2, \text{ if } x \geq -2 \\ 2x+3, \text{ if } x < -2 \end{cases}$, find each of the following:

a) $f(-4) = 2(-4) + 3 = -8 + 3 = \boxed{-5}$

b) $f(-2) = 4(-2)^2 = 4(4) = \boxed{16}$

c) $f(1) = 4 \cdot 1^2 = \boxed{4}$