Honors 125
Homework #2

Name ~~ANSWER KEY~~
Due Wednesday, September 23, 2009

$a=1,\ b=6,\ c=8$

1. On the graph paper below, carefully graph the quadratic function $f(x)=x^2+6x+8$. On your graph indicate both coordinates of the vertex, the y-intercept, and the x-intercept(s), if any.

Vertex: $x=\frac{-b}{2a}=\frac{-6}{2(1)}=-3$

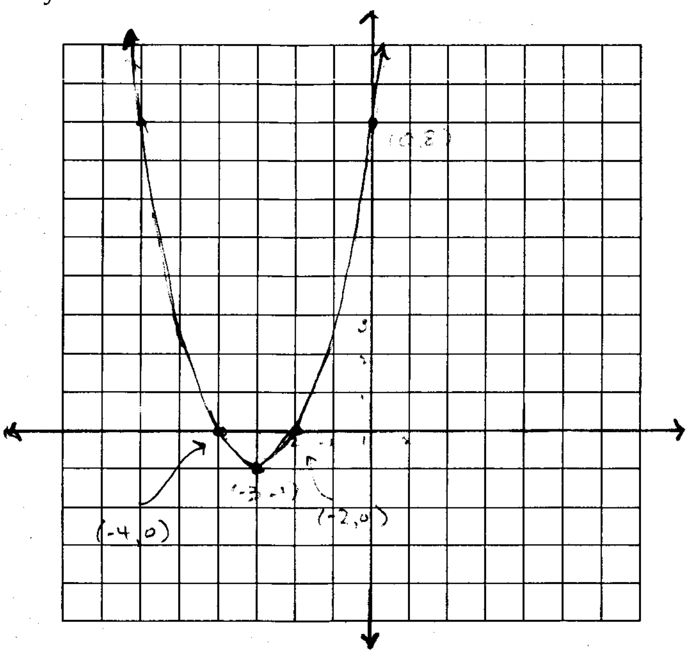
$y=f(-3)=(-3)^2+6(-3)+8=$
$9-18+8=-1$

$\boxed{\text{Vertex: }(-3,-1)}$

y-intercept $=c=f(0)$

$\boxed{y\text{-intercept: }(0,8)}$

5 pts.



x-intercepts:

Let $f(x)=0=x^2+6x+8$
$=(x+4)(x+2)$

$x+4=0 \quad x+2=0$

$x=-4 \quad x=-2$

$\boxed{x\text{-intercepts: } (-4,0),\ (-2,0)}$

10 pts.

2. A tire manufacturer determines that when he sells a certain kind of tire for \$80 per tire, he can sell 600 tires per month, but when he raises the price to \$90 per tire, he can only sell 480 tires per month.

   a) Construct a linear demand function for the tire manufacturer as a function of the selling price p of a tire. $\frac{q_2-q_1}{p_2-p_1}=\frac{600-480}{80-90}=\frac{120}{-10}=-12=m.$ $600=-12(80)+b=$ $960+b$

   $b=1560$

   $\boxed{d(p)=-12p+1560}$ 2

   b) At what price should the manufacturer sell his tires to maximize his **revenue**? 2

Revenue = price × demand $=p(-12p+1560)=-12p^2+1560p$. (Parabola opening down, so maximum revenue occurs at vertex, where $p=\frac{-b}{2a}=\frac{-1560}{2(-12)}=$ $\boxed{\$65 \text{ per tire}}$

   c) The manufacturer further determines that the cost of producing is \$20 per tire, and his fixed costs allocable to tire production are about \$6300 per month. Construct a total cost function for the manufacture of tires as a function of the selling price p of a tire. 2

Variable costs: $20(\#\text{sold})=20\cdot q=20(-12p+1560)$
Fixed costs: \$6300

Total cost $=C(p)=6300-240p+31200$
$=\boxed{-240p+37500=C(p)}$

   d) At what price should the manufacturer sell the tires in order to maximize his **profit**? 2 Profit = Revenue − Total cost $=-12p^2+1560p-(-240p+37500)=$
$-12p^2+1560p+240p-37500=-12p^2+1800p-37500.$ Max @ vertex: $p=\frac{-1800}{2(-12)}=$ $\boxed{\$75 \text{ per tire}}$

   e) For what range of sale prices will the manufacturer make a profit from the sale of tires? 2 Profit function is parabola opening down. The mfgr will make a profit between the x-intercepts: Let $\frac{0}{-12}=\frac{-12p^2+1800p-37500}{-12}$

$0=p^2-150p+3125$
$=(p-25)(p-125).$
$p=25;\ p=125$

May also be done with quadratic formula.

$\boxed{\text{So for } 25<p<125}$ the mfgr makes a profit.

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6 pts. 3. Simplify the expression $\sqrt[4]{81z^8(x^4+y^4)}$, given that x, y, and z are positive real numbers.

$\sqrt[4]{81z^8(x^4+y^4)} = \sqrt[4]{81}\cdot\sqrt[4]{z^8}\cdot\sqrt[4]{x^4+y^4} = \sqrt[4]{3^4}\cdot z^2\cdot\sqrt[4]{x^4+y^4}$

$= 3z^2(\sqrt[4]{x^4+y^4})$

↑ Does not simplify

4. Consider the table below:

x	$f(x)$	$g(x)$	$h(x)$
-2	10	10	16
-1	7	20	8
0	4	40	4
1	1	70	2
2	-2	110	1

increases by 1 each time

3 a) Which, if any, of the functions above could be linear? Justify your answer. Find formula(s) for those function(s).

$7-10 = -3 = 4-7 = 1-4 = -2-1 = -3$ (later minus earlier) so $f(x)$ is linear; equation is $\boxed{y = -3x+4}$ (y-intercept $= b =$ y-value when $x=0$)

3 b) Which, if any, of the functions above could be exponential? Justify your answer. Find formula(s) for those function(s).

$\frac{\text{Later}}{\text{earlier}} = \frac{8}{16} = \frac{1}{2} = \frac{4}{8} = \frac{2}{4} = \frac{1}{2}$; ∴ $h(x)$ is exponential, and $b = 1/2$.

Equation: $h(x) = 4(1/2)^x$ (y-intercept $= A =$ y-value when $x=0$)

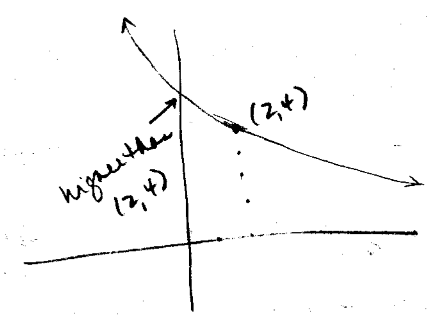
4 5. Find the equation for an exponential function that passes through the points (1, 40) and (2, 10). ① Set up equations: $40 = A\cdot b^1$ $10 = A\cdot b^2$

② Solve for A: $A = \frac{40}{b}$. ③ Insert & solve $10 = \frac{40}{b}\cdot b^2$; $10 = 40b$; $b = \frac{10}{40} = 1/4$

④ Solve for A: $\frac{40\cdot 4}{1/4\cdot 4} = 160$ ⑤ Write equation: $\boxed{y = 160(1/4)^x}$

3 6. Let $f(x) = Ab^x$. Sketch a graph which meets all of the following criteria, or explain why it cannot be done:

- $A \geq 0$ - y-intercept is above x-axis
- $0 < b < 1$ - exponential decay
- The function passes through the point (2,4).



Note: $g(x)$ is NOT linear, because $20-10 \neq 40-20$

NOT exponential because $\frac{40}{20} = 2 \neq \frac{70}{40} = \frac{7}{4}$.

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