Honors 125
Homework #3

Name ANSWER KEY
Due Wednesday, October 14, 2009

Be sure to show all work neatly.

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1. The amount of the antidepressant Prozac, in mg, in the body t hours after taking a standard dose is modeled by the function $P = 20e^{-.019t}$. ($-.019$ is continuous rate)

   1 a) How much Prozac is in a standard dose?
   $\boxed{20 \text{ mg } (= P_0)}$

   2 b) What percent of Prozac leaves the body every hour? Note: $P = 20e^{-.019t} = 20(e^{-.019})^t \approx 20(.9811793622)^t$; so hourly (periodic) rate $= .9811793622 - 1 = -.018820 \approx \boxed{-1.882\%}$ per hour

   2 c) Find the half-life of a dose of Prozac.
   Let $10 = 20(e^{-.019t})$; $\frac{10}{20} = \frac{1}{2} = e^{-.019t}$; $\ln(\frac{1}{2}) = \ln e^{-.019t} = -.019t$; $t = \frac{\ln \frac{1}{2}}{-.019} \approx 36.48$ or about $\boxed{36.5 \text{ hours}}$

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2. In 1990 the population of Mexico was about 84 million people and growing at an annual rate of about 2.6%. (Not a continuous rate)

   1 a) Give a formula for $P(t)$, the population of Mexico (in millions) t years after 1990. $P(t) = 84(1.026)^t$ million

   2 b) What is the **continuous** growth rate of the population of Mexico?
   2.5667746 7% $\ln(1.026) = .0256677467$

   same number

   1 c) Write an equation for the population of Mexico (in millions) t years after 1990 using the continuous growth rate $\boxed{P(t) = 84 \cdot e^{.0256677467t} \text{ million people}} = 84 \cdot e^{\ln 1.026}$ million people

   1 d) If the 1990 growth rate continues, what is the predicted population of Mexico in 2025? (Use either function above.) $(t = 35)$ $84 \cdot e^{.0256677467 \cdot 35} = 84 \cdot (1.026)^{35} = 206.27$ $\approx \boxed{206.3 \text{ million}}$

   2 e) When would the population be expected to reach 100 million people?
   $100 = 84 \cdot (1.026)^t$; $\frac{100}{84} = 1.026^t$; $\ln(\frac{100}{84}) = t \cdot \ln 1.026$; $t = \frac{\ln(\frac{100}{84})}{\ln 1.026} = \boxed{6.79 \text{ yrs}}$ during 1996 (almost 1997)

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3. For the following problems, assume that you know of an investment that pays 4.5% interest per year.

   1 a) Find a model for the value of an investment of \$3500 after t years, if the interest is compounded monthly. $B(t) = 3500(1 + \frac{.045}{12})^{12 \cdot t}$

   1 b) Find the value of the investment above after 5 years. $3500(1 + \frac{.045}{12})^{60} = \$4381.285 \approx \boxed{\$4381.29}$

   1 c) Find a model for the value of an investment of \$3500 after t years, if the interest is compounded continuously. $B(t) = 3500 \cdot e^{.045t}$

   1 d) Find the value of the investment in part c) after 5 years.
   $B(5) = 3500 \cdot e^{(.045)(5)} = \$4383.129 \approx \boxed{\$4383.13}$

   2 e) How long will it take for the investment in part c) to triple in value?

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$\frac{3(3500)}{3500} = \frac{3500 \cdot e^{.045t}}{3500}$; $3 = e^{.045t}$; $\ln 3 = \ln e^{.045t} = .045t$;

$t = \frac{\ln 3}{.045} \approx 24.41$ or about $\boxed{24.4 \text{ years}}$

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4. In 1980 there were about 190 million vehicles (cars and trucks) on the road in the US. The number of vehicles has been growing at a **continuous** rate of 4.5% per year.

1 a) Give a formula for $V(t)$, the number of vehicles (in millions) in the US t years after 1980. $\boxed{190 \cdot e^{.045t} = V(t)}$ $= 190\left(e^{.045}\right)^t$

1 b) What is the **annual percentage** rate of increase in the number of vehicles?
4.6028% $e^{.045} = 1.046028$ $= 190\left[1.046028\right]^t$

1 c) Write an equation for the number of vehicles (in millions) in the US t years after 1980, using the **annual percentage growth rate**.
$190 \cdot (1.046203)^t$

1 d) If the 1980 growth rate continues, what is the predicted number of vehicles on the road in the US in 2020? (Use either function above.)
$t = 40$ 1,149.4 million = 1.149 billion $= 190\left(e^{.045}\right)^{40}$

2 e) In what year would the number of vehicles be expected to reach 250 million?

5. Section 2.3 (page 152), #8, 20, 22, 24, 54.

$$250 = 190\, e^{.045t}$$

$$\frac{250}{190} = e^{.045t}$$

$$\ln\left(\tfrac{25}{19}\right) = .045t$$

$$t = \frac{\ln(25/19)}{.045}$$ = 6.09 yrs later, so during late 1986.

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Section 2.3

#8. $6^{3x+1} = 30$

$\ln 6^{3x+1} = \ln 30$

$(3x+1)(\ln 6) = \ln 30$

$3x+1 = \dfrac{\ln 30}{\ln 6}$

$-1 \qquad -1$

$3x = \dfrac{\ln 30}{\ln 6} - 1$

$x = \frac{1}{3}\left(\dfrac{\ln 30}{\ln 6} - 1\right) \cong \frac{1}{3}(.898244017) = .2994148006 \cong \boxed{.2994}$

(could also have been done with logs:
$x = \frac{1}{3}\left(\dfrac{\log 30}{\log 6} - 1\right)$)

20. $Q = 2000$, when $t=0$; half-life = 5 years.

Let $1000 = 2000 \cdot e^{r \cdot 5}$

$\frac{1}{2} = e^{5r}$; $\ln \frac{1}{2} = \ln e^{5r} = 5r$; $r = \dfrac{\ln \frac{1}{2}}{5} = -.1386294361 \cong -.1386$

so $Q(t) = \boxed{2000 \cdot e^{-.1386t}}$

22. $Q = 2000$ when $t=0$; doubling time = 5.

Let $\dfrac{2(2000)}{2000} = \dfrac{2000 \cdot e^{r \cdot 5}}{2000}$

$2 = e^{5 \cdot r}$; $\ln 2 = \ln e^{5r} = 5r$

$r = \dfrac{\ln 2}{5} \cong .138629$

$\boxed{\text{so } Q(t) = 2000 \cdot e^{.1386t}}$

24. $Q = 1000e^{-.025t}$ (exponential decay, so looking for half-life)

$$\frac{\frac{1}{2}(1000)}{1000} = \frac{1000 \cdot e^{-.025t}}{1000}$$

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$$\tfrac{1}{2} = e^{-.025t}$$

$$\ln(\tfrac{1}{2}) = \ln e^{-.025t} = -.025t$$

$$t = \frac{\ln \frac{1}{2}}{-.025} = 27.72 \sim \boxed{27.7 \text{ years}}$$

54. Strontium-90 has half-life = 28 years.

$\therefore$ if we start with Q_0, in 28 years we have $\frac{1}{2} \cdot Q_0$, so

$$\frac{\frac{1}{2}Q_0}{Q_0} = \frac{Q_0 \cdot e^{-k \cdot 28}}{Q_0} \quad (\text{or } = Q_0 \cdot e^{r(28)})$$

4 $\frac{1}{2} = e^{-28k}$

$$\ln \tfrac{1}{2} = \ln e^{-28k} = -28k$$

$$k = \frac{\ln \frac{1}{2}}{-28} \simeq .024755$$

a) $\boxed{\text{so } Q(t) = Q_0 \cdot e^{-.0248t}}$

b) For $3/5$ of the starting quantity to decay, $2/5$ must be left.

so $$\frac{2/5\, Q_0}{Q_0} = \frac{Q_0 \cdot e^{-.0248t}}{Q_0}$$

$$\frac{2}{5} = e^{-.0248t}$$

$$\frac{\ln(2/5)}{-.0248} = \frac{\ln e^{-.0248t}}{-.0248} = \frac{-.0248t}{-.0248}$$

$$t = \frac{\ln(2/5)}{-.0248} = 36.9 \simeq \boxed{37 \text{ years}}$$