ANSWER KEY

Honors 125, Test 2, Fall 2009

Questions from old tests

1. The amount of cough suppressant A, in mg, remaining in the body t hours after consuming one dose is modeled by the function $A = 10(0.82)^t$.

   a) How much cough suppressant is in the initial dose? (at $t=0$) $\boxed{10 \text{ mg.}}$

   b) What percent of the cough suppressant leaves the body every hour?
   $.82 - 1 = -.18 = \boxed{-18\% \text{ (or 18\% leaves)}}$

   c) What is the continuous rate at which the cough suppressant leaves the body?
   Continuous rate $= \ln(.82) = -.19845\ldots \approx \boxed{-19.85\% \text{ per hour}}$

   d) Find the half-life of cough suppressant in the body.
   easiest to use this function, because no rounding is required. → $5 = 10(0.82)^t$; $\frac{1}{2} = (.82)^t$; $\ln \frac{1}{2} = \ln .82^t = t \ln(.82)$. $t = \frac{\ln(\frac{1}{2})}{\ln(.82)} \approx 3.49$ or $\boxed{\text{about } 3\tfrac{1}{2} \text{ hours}}$

2. The projected population of Japan t years after 2000 is modeled by the function $P(t) = 128e^{-.0020t}$, in millions.

   a) What was the population of Japan in the year 2000? $\boxed{128 \text{ million people}}$

   b) Is the population of Japan expected to grow or decrease? $\boxed{\text{decrease}}$ (because $r_c < 0$)
   ($-.0020$ is a continuous rate of decrease)

   c) What is the expected annual rate of change of the population of Japan?
   $e^{-.0020} \approx .998002 = -.001998 = -0.1998\%$ (virtually identical to continuous)

   d) What is the expected population in 2030?
   Note: I used the function that required no rounding. → $128 \cdot e^{-.0020(30)} = 120.54 \sim \boxed{120.5 \text{ million people}}$

   e) When would the population of Japan be expected to reach 110 million?
   → $110 = 128 \cdot e^{-.0020t}$; $\frac{110}{128} = e^{-.0020t}$; $\ln\left(\frac{110}{128}\right) = -.0020t$; $t = \frac{\ln(\frac{110}{128})}{-.0020} \approx 75.77$ yrs $\boxed{\text{(so during 2075, or by 2076)}}$

3. Solve for x:

   a) $350 = 280 + 4e^{-x}$
   $-280 \quad -280$
   $70 = 4e^{-x}$; $\frac{70}{4} = 17.5 = e^{-x}$ ← isolate exponential term ; $\ln 17.5 = -x$
   $\boxed{x = -\ln 17.5 \approx -2.8622\ldots}$ either is fine

   b) $90 = 15 \cdot 7^{2x}$ (you may leave your answer either in exact form or a decimal approximation.) $\frac{90}{15} = 6 = 7^{2x}$ ← isolate exponential term ; $\log 6 = \log 7^{2x} = 2x \log 7$; $\boxed{x = \frac{\log 6}{2 \log 7} \approx .46039\ldots \sim .4604}$
   (could also have been done with ln)

4. Find each of the following limits, if the limit exists. If it does not exist, write DNE and briefly explain your answer.

   a) $\lim_{x \to 2} \frac{x^2 - 3x + 2}{x + 2} = \frac{0}{4} = \boxed{0}$
   plug in: $\frac{4-6+2}{4} = \frac{0}{4}$

   b) $\lim_{x \to 1} \frac{x+1}{x^2 + 5x - 6}$ $\boxed{\text{DNE}}$ (vertical asymptote)
   plug in: $\frac{1+1}{1+5-6} = \frac{2}{0}$

   c) $\lim_{x \to 1} \frac{x-1}{x^2 + 5x - 6} = \lim_{x \to 1} \frac{\cancel{x-1}}{\cancel{(x-1)}(x+6)} = \lim_{x \to 1} \frac{1}{x+6} = \boxed{\frac{1}{7}}$
   plug in: $\frac{1-1}{1+5-6} = \frac{0}{0}$: keep going

$$= \lim_{x \to \infty} \frac{\frac{x}{x^2} - \frac{1}{x^2}}{\frac{x^2}{x^2} + \frac{5x}{x^2} - \frac{6}{x^2}} = \lim_{x \to \infty} \frac{\frac{1}{x} - \frac{1}{x^2}}{1 + \frac{5}{x} - \frac{6}{x^2}} = \frac{0}{1} = \boxed{0}$$

d) $\lim_{x \to \infty} \dfrac{x-1}{x^2+5x-6}$ $= 0$ (from comparison of exponents in numerator & denominator)

5. Problem 29, page 194 (limits from a graph). (answers are in book)

Problem 30: a) DNE $(+\infty)$, b) 0, c) DNE $(+\infty)$, d) 0, e) 0, f) 1. (discontinuous at both $x=0$ and $x=1$)

6. True or False. Indicate whether each statement is true or false. If the statement is true, explain how you know. If it is false, give a counterexample.

a) __F__ If a function is not continuous at a point, then it is not defined at that point.

Counter-example: Let $f = \begin{cases} x^2, & \text{if } x \neq 0 \\ 4, & \text{if } x = 0 \end{cases}$. f is not continuous at $x=0$, but it is defined there.

b) __T__ If $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x) = 2$, then $\lim_{x \to 1} f(x) = 2$. (definition of 2-sided limit)

2 sided limit

c) __F__ If $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x) = 2$, then $f(1) = 2$. ← function value only. The one-sided limits may exist at a point even if the function is undefined at the point. (e.g. let $f(x) = \frac{x^2-1}{x-1}$. Then $f(1)$ DNE, but $\lim_{x \to 1} f(x) = 2$)

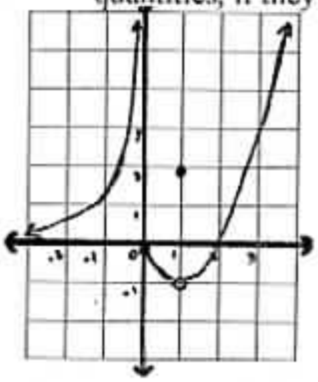
7. Find all points of discontinuity of the function $f(x) = \begin{cases} \frac{1}{x}, & \text{if } x < 0 \\ 2x+1, & \text{if } 0 \le x \le 2 \\ x^2+1, & \text{if } x > 2 \end{cases}$ Possible discontinuities at $x=0$, $x=2$

(NOTE: Be sure to look at similar problems in section 3.3, # 43-50).

Check: $\lim_{x \to 0^-} \frac{1}{x}$ DNE. ∴ $\boxed{\text{discontinuous at } x=0.}$ $\lim_{x \to 2^-} (2x+1) = 2 \cdot 2 + 1 = 5 = f(2)$; $\lim_{x \to 2^+} x^2+1 = 2^2+1 = 5$

Because $f(2) = \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = 5$, continuous at $x=2$.

8. The graph of the function f is given below. Use the graph to compute the following quantities, if they exist. If they do not exist, write DNE and explain why.



a) $\lim_{x \to 0^+} f(x)$ __0__

b) $\lim_{x \to 0^-} f(x)$ __DNE__ (vertical asymptote) $(+\infty)$

c) $\lim_{x \to 0} f(x)$ __DNE__ (because no 2-sided limit)

d) $\lim_{x \to 1} f(x)$ __−1__ (value being approached from both sides)

e) $f(1)$ __2__ (value of "dot")

f) What value, if any, should be assigned to $f(1)$ to make $f(x)$ continuous at $x = 1$? Let $f(1) = -1$ – "dot" filled in, and the function is continuous there.

g) What value, if any, should be assigned to $f(0)$ to make $f(x)$ continuous at $x = 0$? None. Because the limit doesn't exist at $x=0$, the function can't be made continuous.

9. Consider the function $f(x)=\begin{cases}2x-1, & \text{if } x>-1^{+}\\ x^{2}-4, & \text{if } x<-1^{-}\\ 0, & \text{if } x=-1\end{cases}$

a) Showing all work, find each of the following, if it exists. Otherwise write DNE and briefly explain your answer.

i) $\lim_{x\to -1^{+}} f(x)$ $= \lim_{x\to -1} (2x-1) = 2(-1)-1 = \boxed{-3}$

ii) $\lim_{x\to -1^{-}} f(x)$ $\lim_{x\to -1^{-}} (x^2-4) = (-1)^2-4 = 1-4 = \boxed{-3}$

iii) $\lim_{x\to -1} f(x)$ $= -3$ (because 1-sided limits are equal)

iv) $f(-1)$ 0

b) Is f continuous at $x=-1$? No Why or why not?

Because $\lim_{x\to -1} f(x) = -3 \neq f(-1) = 0$.

c) Find $\lim_{x\to 3} f(x)$, if it exists. $\lim_{x\to 3} 2x-1 = 6-1 = \boxed{5}$

d) Is f continuous at $x=3$? yes Why or why not? $2x-1$ is a linear function, continuous everywhere. Only possible discontinuity is at $x=-1$, at break in piecewise defined function.

10. Consider the function $f(x)=\begin{cases}2x-1, & \text{if } x>1^{+}\\ 5x^{2}-2, & \text{if } x<1^{-}\\ 4, & \text{if } x=1\end{cases}$ Showing all work, do the following:

a) Find $\lim_{x\to 1^{+}} f(x)$, if it exists. $\lim_{x\to 1^{+}} 2x-1 = 2-1 = \boxed{1}$

b) Find $\lim_{x\to 1^{-}} f(x)$, if it exists. $\lim_{x\to 1^{-}} 5x^2-2 = 5-2 = \boxed{3}$

c) Find $\lim_{x\to 1} f(x)$, if it exists. DNE (because 1-sided limits are different)

d) Find $f(1)$, if it exists. 4

e) Is f continuous at $x=1$? No Why or why not? No limit there.

f) Find $\lim_{x\to 4} f(x)$, if it exists. $\lim_{x\to 4} 2x-1 = 8-1 = 7$

g) Is f continuous at $x=4$? yes Why or why not? (see d above)

2. Let $f(x)=\dfrac{2x-6}{x^{2}-9}$.

If $f(3) = 2/6 = 1/3$, the function will be continuous

a) Determine what value, if any, to assign to $f(3)$ to make f continuous at $x=3$. If there is none, briefly explain why. find $\lim_{x\to 3} \frac{2x-6}{x^2-9} = \lim_{x\to 3} \frac{2(\cancel{x-3})}{(x+3)(\cancel{x-3})} = \lim_{x\to 3} \frac{2}{x+3} = \boxed{\frac{2}{6}}$ at $x=3$.

b) Determine what value, if any, to assign to $f(-3)$ to make f continuous at $x=-3$. If there is none, briefly explain why. $\boxed{\text{NONE}}$

find $\lim_{x\to -3} \frac{2x-6}{x^2-9}$ = DNE. Because the limit doesn't exist, $f(x)$ can't be made continuous at $x=-3$.

plug-in: $\frac{-6-6}{(-3)^2-9} = \frac{-12}{0}$ } limit doesn't exist.