Answer Key

Honors 125 **Fall 2009**

Test 1 Problems from old tests

1. Find the slope-intercept form of the equation of the line through the points (4, 3) and (5, 1). $m = \frac{3-1}{4-5} = \frac{2}{-1} = -2$; $3 = -2(4)+b = -8+b$, $b = 11$; so: $y = -2x + 11$

2. Find the slope-intercept form of the equation of the line through the point (10, -2.5) and decreasing at a rate of 5 units of *y* direction per 4 units of *x*. $m = -5/4$

$-2.5 = -5/4(10)+b = -17.5+b$ $b = 10$; so $y = -5/4\,x + 10$

3. Solve for *x*:
   a) $2x + 3x^2 = 0$
   $x(2+3x) = 0$; $\boxed{x=0}$, $2+3x=0$; $3x=-2$; $\boxed{x = -2/3}$
   b) $x^2 + x = 12$
   $x^2+x-12=0$ $(x+4)(x-3)=0$; $\boxed{x=-4,\ x=3}$
   c) $2x^2 + x - 5 = 0$ Use quadratic formula: $x = \frac{-1 \pm \sqrt{1-4(2)(-5)}}{2\cdot 2} = \frac{-1 \pm \sqrt{1+40}}{4} = \boxed{\frac{-1 \pm \sqrt{41}}{4}}$

4. A stone is thrown up into the air off of a tall building. It rises and then falls to the ground. The height, *H*, (in feet) of the stone above ground is a function of the elapsed time *t* in seconds since the stone was thrown, and is given by the function
$H(t) = -16t^2 + 48t + 98$

   a) Find the vertical intercept and explain what it represents in terms of height and time.
   $H(0) = 98$ = vertical intercept. Interpretation: the stone was thrown from a starting height of 98 feet.
   b) Find the horizontal intercept(s) and explain what it/they represents in terms of height and time.
   Let $-16t^2+48t+98=0$. Then $t = \frac{-48 \pm \sqrt{48^2-4(-16)(98)}}{2(-16)} = \frac{-48 \pm \sqrt{8576}}{-32} = \frac{-48 \pm 92.61}{-32}$ = $t = -1.39$ (not in domain) or $t = 4.39$ ← time in seconds when stone hits the ground.
   c) Find the vertex (both coordinates) and explain what it represents in terms of height and time. $\frac{-b}{2a} = -\frac{48}{2(-16)} = \frac{-48}{-32} = \frac{3}{2} = 1.5$ seconds (time to reach highest point) $H(1.5) = 134$ ft = highest point stone reaches

5. Sales figures showed that your company sold 1960 pen sets each week when they were priced at \$3 per pen set and 1800 pen sets when they were priced at \$5 per pen set. Points are (3, 1960), (5, 1800) Slope = $m = \frac{1960-1800}{3-5} = \frac{160}{-2} = -80$. $1800 = -80(5)+b$. $b = 1800+400 = 2200$

Points: (price, quantity)

   a) What is the linear demand function for your pen sets, where *p* is the price per pen set and *q*(*p*) is the number of pen sets that you can sell? $q(p) = -80p + 2200$.
   b) What do the *p*- and *q*- intercepts of your function represent? $\boxed{q \text{ intercept} = 2200: \text{\# that can be given away } (p=0)}$
   p intercept (let $0 = -80p+2200$; $2200 = 80p$; $p = 27.5$) at \$27.50/set, none will sell.
   c) What does the slope represent?
   For each \$1.00 increase in price, 80 fewer sets will sell.
   d) Find the function that models the weekly revenue received from the sale of pen sets at a price *p* per set. $R(p) = (\text{price})(\text{quantity}) = p(-80p + 2200)$ $= \boxed{-80p^2 + 2200p = R(p)}$
   e) At what price should the pen sets be sold to maximize revenue?

Find vertex: $p = \frac{-b}{2a} = \frac{-2200}{2(-80)} = \frac{-2200}{-160} = \boxed{\$13.75/\text{set}}$

f) What is the maximum weekly revenue that can be generated by the sale of pen sets? (Use Revenue function): $R(13.75) = 13.75(-80(13.75)+2200) = 13.75(1100) = \boxed{\$15{,}125.00}$

g) How many pen sets must be sold to generate that revenue?
(Use linear demand function): $q(13.75) = -80(13.75) + 2200 = \boxed{1100 \text{ pen sets}}$

h) If each pen set costs \$1.50 to produce, find a function that models the profit that can generated by the sale of pen sets.

Total cost: \$1.50 × (quantity) $= 1.50(-80p+2200) = -120p + 3300$; Profit = Revenue − cost $= -80p^2 + 2200p - (-120p + 3300) = \boxed{-80p^2 + 2320p - 3300}$

i) At what sale price of pen sets is profit maximized?

At vertex: $\frac{-b}{2a} = p = \frac{-2320}{2(-80)} = \frac{-2320}{-160} = \boxed{\$14.50 \text{ per set.}}$

6. **Supply/demand.**

Note: points are (price, quantity)

a) A bike shop can sell 80 novice mountain bikes per month at \$300 per bike, and it can sell 40 bikes per month at \$400 per bike. Points (300, 80), (400, 40)

i) What is the linear demand function for bike sales by the bike shop?

$m = \frac{80-40}{300-400} = \frac{40}{-100} = -\frac{2}{5} = -.4$. $80 = -\frac{2}{5}(300) + b = -120 + b$; $b = 200$. $\boxed{q = -2/5\, p + 200}$

ii) What is the significance of the slope that you found above? (Be sure to specify units.) For every \$5 increase in price, 2 fewer bikes are sold (or for every \$10 increase, 4 fewer are sold.)

Points are: (price, quantity) → b) The producer of the bikes is willing to supply 50 bikes each month if the bikes are sold at \$300 each and 70 each month if they are sold at \$400 each. points: (300, 50), (400, 70)

i) What is the linear supply function for the bikes? $m = \frac{70-50}{400-300} = \frac{20}{100} = 1/5$; $50 = 1/5(300) + b = 60 + b$. $b = -10$

$\boxed{S(p) = \frac{1}{5}p - 10}$

ii) What is the significance of the slope of the supply function you found? (Be sure to specify units.) For each \$5 increase in price, the producer will supply 1 additional bike.

c) At what price should the bikes be sold so that there is neither a shortage nor a surplus? Let $d(p) = s(p)$ and solve: $-2/5\,p + 200 = 1/5\,p - 10$. (Multiply by 5:
$-2p + 1000 = p - 50$ $1050 = 3p$. $\boxed{p = \$350}$

Points: (price, quantity) 7. A bookstore is selling study guides prepared for it by faculty members. It determined that when the study guides were priced at \$25 each, only 20 were sold per month, but when they were priced at \$10 each, 80 copies were sold per month. Points: (25, 20) (10, 80)

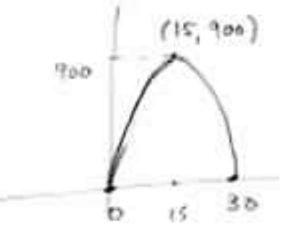
a) Find the linear demand function for study guides in terms of the price per copy.

$m = \frac{80-20}{10-25} = \frac{60}{-15} = -4$. $80 = -4(10) + b = -40 + b$. $b = 120$. $\boxed{q(p) = -4p + 120}$

b) Express the bookstore's total revenue from the sale of study guides, R, as a function of the price per copy.

$R(p) = p \cdot q = p(-4p + 120) = \boxed{-4p^2 + 120p = R(p)}$

c) Graph the resulting revenue function, labeling at least three points.



x-intercepts: let $p(-4p+120) = 0$
$p = 0$; $-4p + 120 = 0$; $p = 30$.

vertex: $-\frac{b}{2a} = \frac{-120}{2(-4)} = \frac{-120}{-8} = 15$. $R(15) = 900$

d) At what price should the study guides be sold to maximize revenue?

$\$15.00 = p$

e) What is the largest possible revenue?

$R(15) = \$900$

8. Consider the following chart of values.

Changes +1 in x-direction

x	$F(x)$	$G(x)$	$H(x)$
-2	1	1	1
-1	3	3	3
0	6	5	9 ← y-intercept
1	10	7	27
2	15	9	81

a) Which set of values, if any, could represent a linear function? G(x)
Briefly justify your answer and write an equation for each linear function.
$\frac{3-1}{1} = \frac{5-3}{1} = \frac{7-5}{1} = \frac{9-7}{1} = 2 = m$ $\quad G(x) = 2x+5$
(constant difference between successive terms.)

b) Which set of values, if any, could represent an exponential function?
H(x) Briefly justify your answer and write an equation for each exponential function. $\frac{3}{1} = \frac{9}{3} = \frac{27}{9} = \frac{81}{27} = 3 = b$ $\quad \boxed{H(x) = 9 \cdot 3^x}$
(constant ratio of successive terms)

c) If any set of values is neither exponential nor linear, explain why. F(x)
Not linear, because $3-1 \neq 6-3$ (difference isn't constant). Not exponential because $\frac{3}{1} = 3 \neq 6/3$ (ratio isn't constant.)

9. The population of pandas in captivity in January 2006 was about 188 pandas.

a) During the summer of 2006 a record number of panda babies was born and survived, for a net increase of 16 pandas in captivity during the year. Write a function expressing the population of pandas t years from January 2006, if the population of pandas in captivity continues to grow by the **same number** of pandas per year. $\boxed{P(t) = 188 + 16t}$ — linear growth

b) Write a function expressing the population of pandas t years from January 2006, if the population of pandas in captivity continues to grow at the same annual **percentage rate** as 2006. % increase is $16/188 \cong .085$; so $P(t) = P_0(1+r)^t = \boxed{188(1.085)^t = P(t)}$
OR: # in 2007 = 188 + 16 = 204; $b = \frac{204}{188} = 1.085$; so $P(t) = 188(1.085)^t$

c) If the number of pandas continues to grow according to the function you found in part a), how many pandas will there be at the beginning of 2015? 332 pandas
$t = 15$, so $P(9) = 188 + 16(9) = 332$

d) If the population of pandas grows according to the function you found in part b), how many pandas will there be at the beginning of 2015? 392 pandas
$188(1.085)^9 \cong 391.7 \cong 392$

10. At time t, in hours, after taking a pain killer, the amount A, in mg, remaining in the body is given by $A = 400(0.56)^t$.

a) What was the initial amount taken? 400 mg $(= Q_0 = A)$

b) What percent leaves the body every hour?
$b = 1+r$; so $r = b - 1 = .56 - 1 = -.44$. So 44% leaves (or −44%)

c) How much of the pain killer is left after 6 hours?
$400(0.56)^6 \cong 12.34$ mg.

d) According to this model, when will the pain killer be completely gone? Never
(exponential decay approaches but does not reach 0)

11. The projected population of Japan t years after 2000 is modeled by the function $P(t) = 128(.998)^t$, in millions.

a) What was the population of Japan in the year 2000? 128 million people

b) Is the population of Japan expected to increase or decrease? decrease (because $b < 1$)

c) What is the expected annual rate of change of the population of Japan?

$r = b - 1 = .998 - 1 = -.002 = -0.2\%$ (or 0.2% decrease)

d) Using this model, what is the expected population in 2030? ($t = 30$)

$128(.998)^{30} \approx 120.5$ million people.

12. Find an equation for the exponential function that passes through the points (1, 4) and (3, 36). use $y = A \cdot b^x$

I. $4 = A \cdot b^1$ $36 = A \cdot b^3$ II. $A = \frac{4}{b}$ III. $36 = \frac{4}{b} \cdot b^3 = 4b^2$; $b^2 = 9$; $b = 3$.

$A = 4/3$. $\boxed{\text{So } f(x) = 4/3 \cdot 3^x}$

13. Find an equation of an exponential function that passes through the points (4, 3) and (5, 1).

I. $3 = A \cdot b^4$ $1 = A \cdot b^5$

II. $A = \frac{3}{b^4}$ III. $1 = \frac{3}{b^4} \cdot b^5 = 3b$; $b = 1/3$; $A = \frac{3}{(1/3)^4} = 3^5 = 243$; $\boxed{f(x) = 243 \cdot (1/3)^x}$

14. The population of North Atlantic right whales was estimated to be about 300 whales at the beginning of the year 2000. At the beginning of 2003 the number of whales was estimated to be about 275 whales.

a) Find an exponential model for the population of North Atlantic right whales t years after the beginning of 2000, if the rate of decline continues unchanged. (at $t = 0$, $P_0 = 300$, at $t = 3$, $P(t) = 275$)

Use: $P(t) = P_0 \cdot b^t$. $275 = 300 \cdot b^3$; $b^3 = \frac{275}{300}$; $b = \sqrt[3]{\frac{275}{300}} \approx .9714$. $\boxed{P(t) = 300(.9714)^t}$

b) If the estimates of population behavior above are correct, what is the expected population of North Atlantic right whales in 2030? ($t = 30$)

$P(30) = 300(.9714)^{30} = 125.6 \approx 126$ whales

c) If the decline in population is linear rather than exponential and continues linearly at the same rate as between 2000 and 2003, find a functional model for the population of North Atlantic right whales t years after the beginning of 2000. $m = \frac{275 - 300}{3} = \frac{-25}{3}$

$\boxed{\text{so } P(t) = \frac{-25}{3}t + 300}$ $b = 300$

d) Using the linear model, what is the expected population of North Atlantic right whales in 2030? $P(30) = -\frac{25}{3}(30) + 300 = -250 + 300 = \boxed{50 \text{ whales}}$

e) When will the population of right whales reach 0 under the linear model? by beginning of 2036.

Let $-\frac{25}{3}t + 300 = 0$. $-25t + 900 = 0$; $900 = 25t$; $t = 36$

f) When will the population of right whales reach 0 under the exponential model? Never (exponential decay approaches but does not reach 0)

15. A standard dose of the drug Imitrex is 6 mg. The amount of Imitrex in the bloodstream decays exponentially, with half removed every 2 hours (in other words, it has a half-life of 2 hours).

a) Find an exponential decay model for the amount of Imitrex in the bloodstream t hours after taking a standard dose. use: $Q(t) = Q_0 \cdot b^t$; $3 = 6(b)^2$; $b^2 = 1/2$; $b = \sqrt{1/2}$; so $\boxed{Q(t) = 6 \cdot (\sqrt{1/2})^t}$

b) What percent of the Imitrex leaves the bloodstream every hour? $\sqrt{1/2} = b \approx .707$

$r = b - 1 = .707 - 1 = -.293 = \boxed{-29.3\%}$ (or 29.3% leaves each hour)

c) How much Imitrex is left in the bloodstream after 9 hours?

$Q(9) = 6(\sqrt{1/2})^9 = .265 \sim .27$ mg.